Quantum State Measurement of Optical Fields and Ultrafast Statistical Sampling

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OUTLINE

PART 1

1. Noise Properties of Photodetectors

2. Quantization of Light

3. Direct Photodetection and Photon Counting

PART 2

4. Balanced Homodyne Detection

5. Ultrafast Photon Number Sampling

PART 3

6. Quantum State Tomagraphy

PART 3

QUANTUM STATE TOMOGRAPHY

For introduction, see:

M.R., Contemp. Physics 38, 343 (1997)

Niels Bohr:

"In quantum physics ... evidence about atomic objects obtained by different experimental arrangements exhibits a novel kind of complementarity relationship. ... Such evidence, which appears contradictory when combination into a single picture is attempted, exhausts all conceivable knowledge about the object. ...

Moreover, a completeness of description like that aimed at in classical physics is provided by the possibility of taking every conceivable experimental arrangement into account."

from "Quantum Physics and Philosophy" (1958), reprinted in *Niels Bohr Collected Works* , *Foundations of Quantum Physics II, Vol.7*, ed. by J. Kalckar (Elsevier, Amsterdam, 1996).

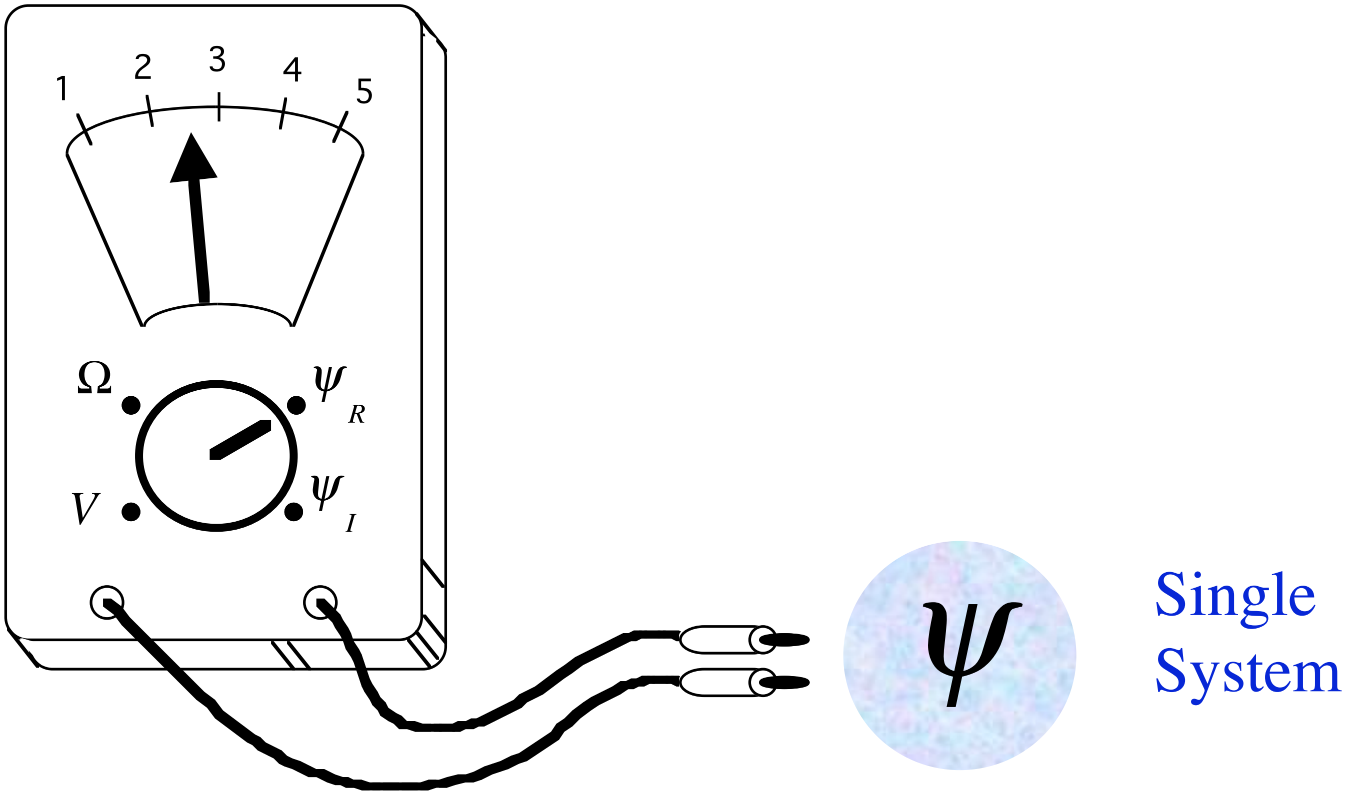
"To know the quantum mechanical state of a system implies, in general, only statistical restrictions on the results of measurements."

-- John S. Bell, 1966

Converse--

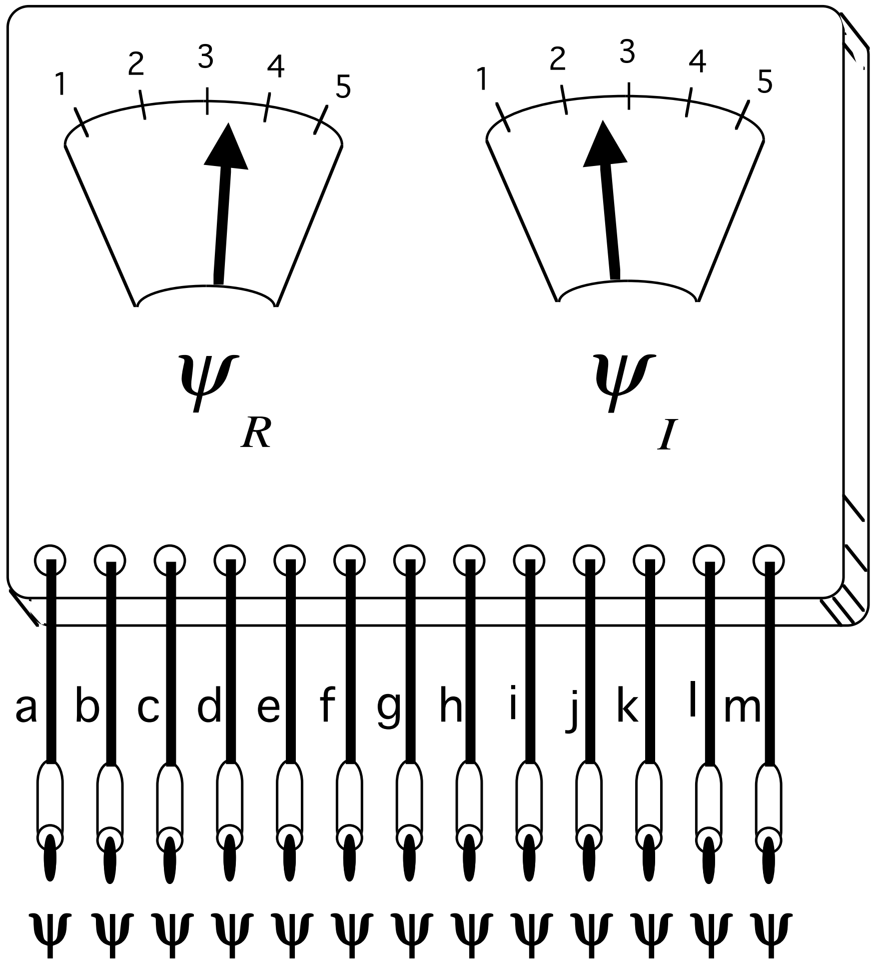
To know the statistical restrictions on the results of measurements is to know the quantum mechanical state of a system.

Volt - Ohm - Psi - Meter Pat. Pend.



The state of an *individual* system <u>*cannot*</u>, even in principle, be measured. The state of an ensemble of identically prepared systems can be measured.

A Realistic Psi - Meter



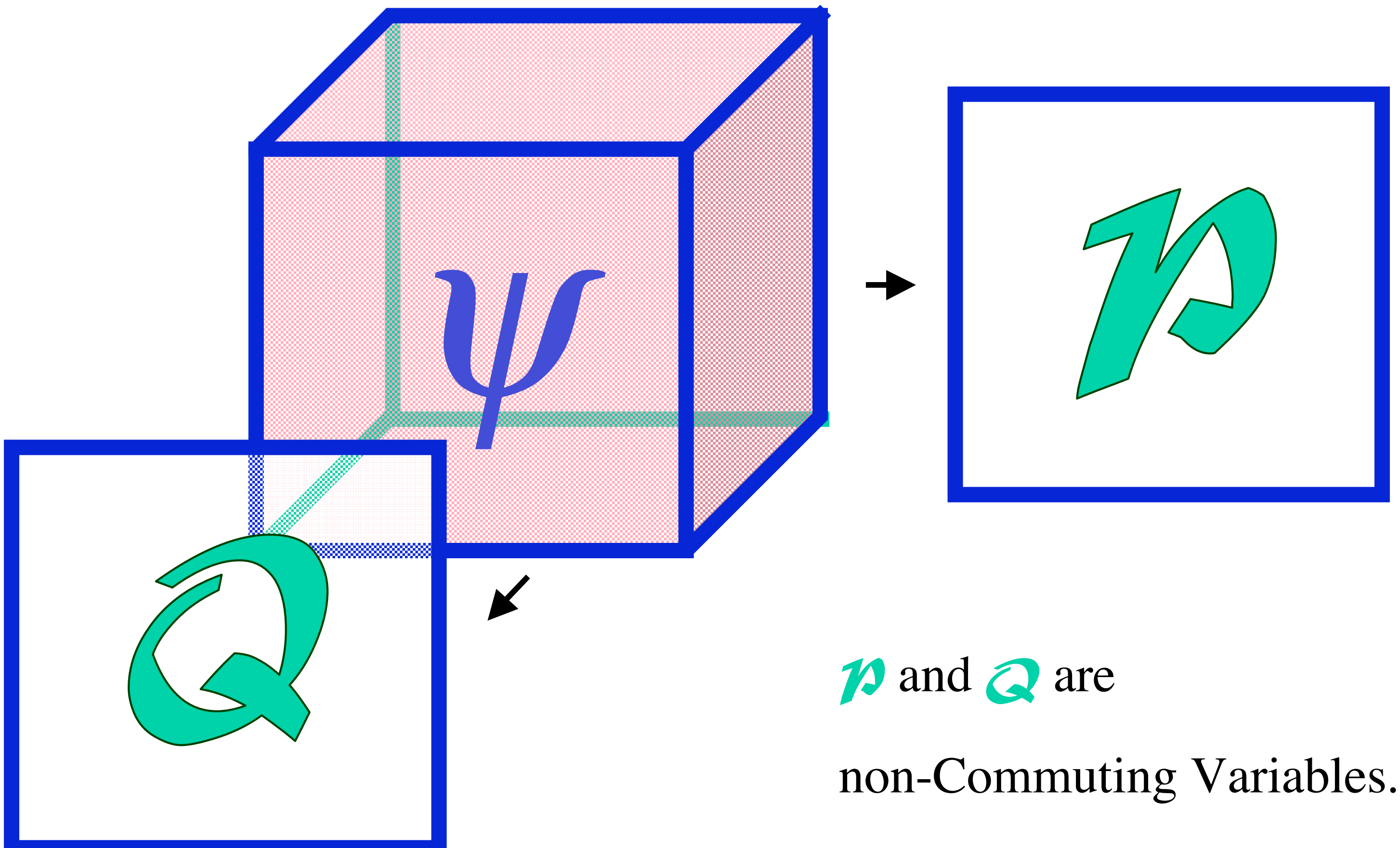
$$|\psi\rangle = \sum_j a_j |\phi_j\rangle$$

A "Quorum" of Variables

A PHYSICAL EMSEMBLE OF SYSTEMS

M.R., Contemp. Physics 38, 343 (1997)

Quantum States Provide Information Security

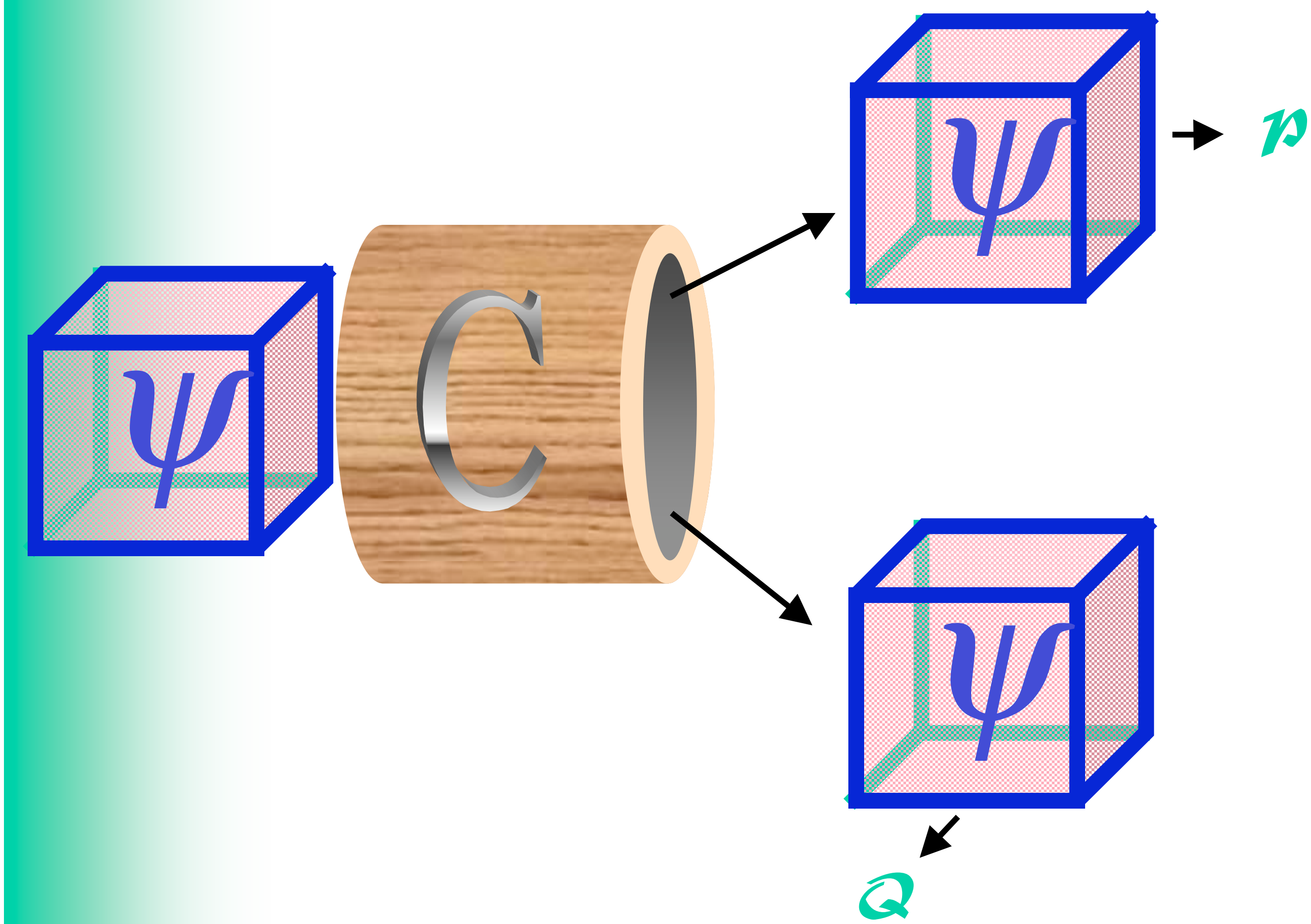


P and Q are

non-Commuting Variables.

Can measure only one.

Quantum-State Cloning?



$\mathcal{P}$ and $\mathcal{Q}$ are

Both determined?

No - Cloning Theorem (Wootters and Zurek)

$$|\psi\rangle_{SYS}|\chi_{ref}\rangle_{AUX} \xrightarrow{C} |\psi\rangle_{SYS}|\psi\rangle_{AUX} \quad ?$$

(basis state)

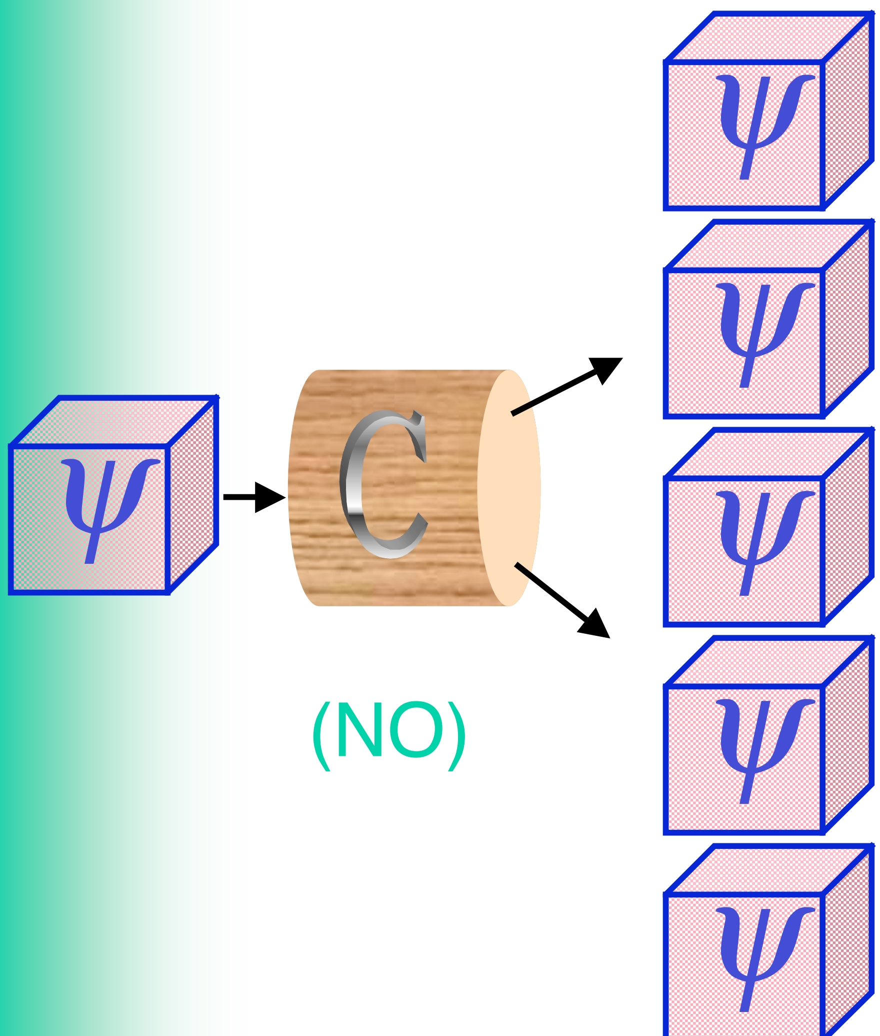
$$|\phi_J\rangle_{SYS}|\chi_{ref}\rangle_{AUX} \xrightarrow{C} |\phi_J\rangle_{SYS}|\phi_J\rangle_{AUX}$$

(arbitrary state)

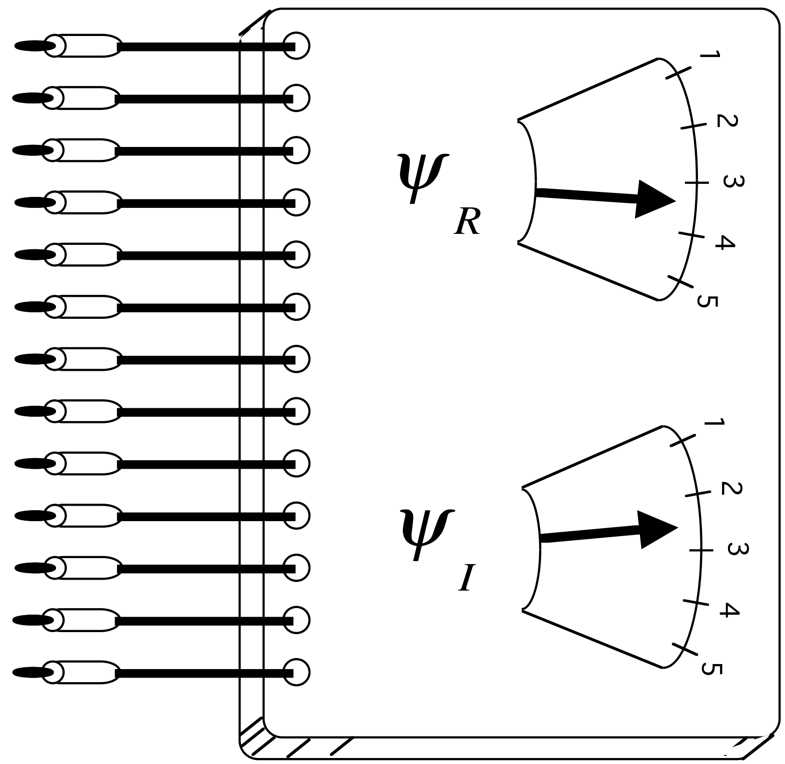
$$\left(\sum_j a_j\,|\phi_j\rangle_{SYS}\right)|\chi_{ref}\rangle_{AUX} \xrightarrow{C} \left(\sum_j a_j\,|\phi_j\rangle_{SYS}\,|\phi_j\rangle_{AUX}\right)$$

$$\neq \left(\sum_j a_j\,|\phi_j\rangle_{SYS}\right)\left(\sum_j a_j\,|\phi_j\rangle_{AUX}\right)$$

No Cloning <---> No State Measurement of Single Particle



A Quorum



Systems measured include:

Optical squeezed states: D. T. Smithey, M. Beck, M. G. Raymer, and A. Faridani, Phys. Rev. Lett. **70**, 1244 (1993). G. Breitenbach, S. Schiller, and J. Mlynek, Nature **387**, 471 (1997). K. Banaszek, C. Radzewicz, K. Wodkiewicz, and J. S. Krasinski, Phys. Rev. A **60**, 674 (1999).A. I. Lvovsky, H. Hansen, T. Aichele, O. Benson, J. Mlynek, and S. Schiller, Phys. Rev. Lett. **87**, 050402 (2001).

Polarization states of photon pairs: A. G. White, D. F. V. James, P. H. Eberhard, and P. G. Kwiat, Phys. Rev. Lett. **83**, 3103 (1999).

Angular momentum states of electrons: J. R. Ashburn, R. A. Cline, P. J. M. van der Burgt, W. B. Westerveld, and J. S. Risley, Phys. Rev. A **41**, 2407 (1990).

Molecular vibrational states: T. J. Dunn, I. A. Walmsley, and S. Mukamel, Phys. Rev. Lett. **74**, 884 (1995).

Motional states of trapped ions: D. Leibfried, D. M. Meekhof, B. E. King, C. Monroe, W. M. Itano, and D. J. Wineland, Phys. Rev. Lett. **77**, 4281 (1996).

Motional states of atomic beams: C. Kurtsiefer, T. Pfau, and J. Mlynek, Nature **386**, 150 (1997).

Nuclear spin states: I. L. Chuang, N. Gershenfeld, M. G. Kubinec, and D. W. Leung, Proc. R. Soc. London A **454**, 447 (1998).

Physics and Astronomy Classification Scheme (**PACS**) code:
03.65.Wj, State Reconstruction and Quantum Tomography

Why Measure the State?

Once the state is obtained, distributions or moments of quantities can be calculated, even though they have not been directly measured (or are not measurable, in principle).

Examples -- The ability to measure photon number distributions at the single-photon level, even though the detectors used had noise levels too large to directly measure these distributions.

-- Distributions of optical phase have been determined, even though there is no known experimental apparatus capable of directly measuring this phase.

-- Expectation values of the number-phase commutator have been measured, even though this operator is not Hermitian, and thus cannot be directly observed.

Even if the full quantum state is not measured, performing many measurements corresponding to different observables can yield information about optical fields. (Linear Optical Sampling)

-- For example, one can obtain information, with high time-resolution (on the time scale of 10's of *fs*), on the photon statistics of light –

propagation in scattering media ,

light emitted by pulsed lasers,

performance of optical communication systems.

QUANTUM STATE TOMOGRAPHY

The state of an ensemble of identically prepared systems can be measured.

- Each member of an ensemble of systems is prepared by the same state-preparation procedure.
- Each member is measured only once, and then discarded (or recycled).
- A mathematical transformation is applied to the data in order to reconstruct, or infer, the state of the ensemble.

- A single member of the ensemble is described by the state, but the state cannot be determined by measuring only that individual.

PURE STATES AND MIXED STATES

How is the state of a quantum system represented?

- Pure State Case -- Every system is prepared using exactly the same procedure. Described by

$$|\psi\rangle, \; or \;\; \langle x|\psi\rangle \;=\; \psi(x)$$

- Mixed State Case -- Each system is prepared using one of many procedures, with probability $P(\psi)$. The density operator is an ensemble average

$$\hat{\rho} = \sum_{\psi} P(\psi)\,|\psi\rangle\langle\psi|$$

or, in wave-function representation:

$$\rho(x,x') = \sum_{\psi} P(\psi)\;\psi(x)\psi^*(x') = \langle \psi(x)\psi^*(x')\rangle_{ens}$$

QUANTUM STATE TOMOGRAPHY

How many different variables need to be statistically characterized? (For a one-dimensional system)

• Pure State Case -- Often it is sufficient to characterize just two variables. (for example: q, p)

• Mixed State Case -- Need to characterize many variables.
(for example: q, p, $q+p$, $q-p$, $q\cos\theta + p\sin\theta$, etc).

State Reconstruction Schemes

1. Limit of infinite amount of perfect data: Deterministic inversion of data to yield density matrix. (Here we restrict ourselves to this case.)

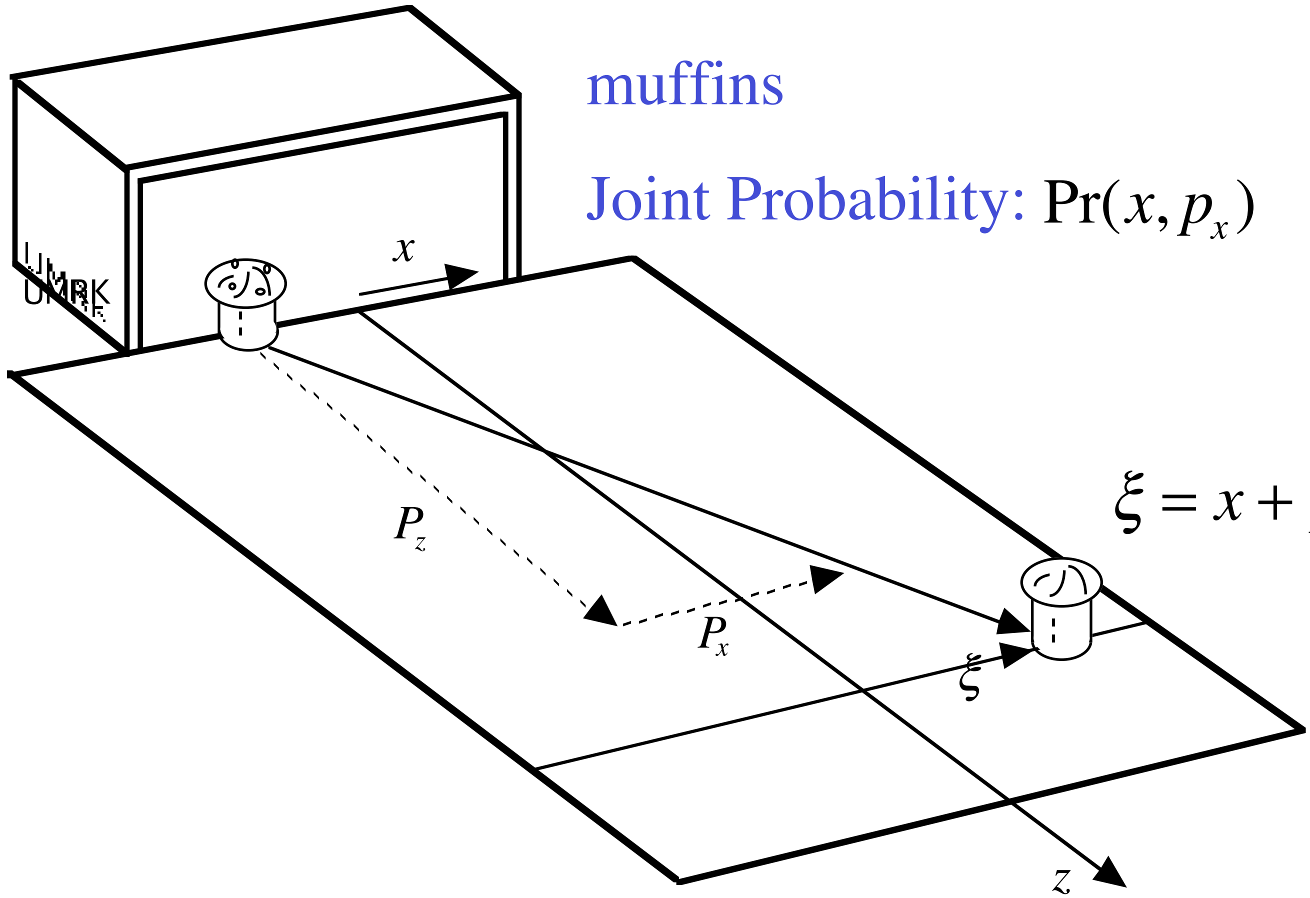
2. Finite amount of noisy data: Estimation of the density matrix by:

- least-squares estimation
- maximum-entropy estimation
- maximum-likelihood estimation

How to Determine a Statistical state of an Ensemble of *Classical Particles* if you are allowed to measure only one variable (say position x, but not momentum p_x)?

muffins

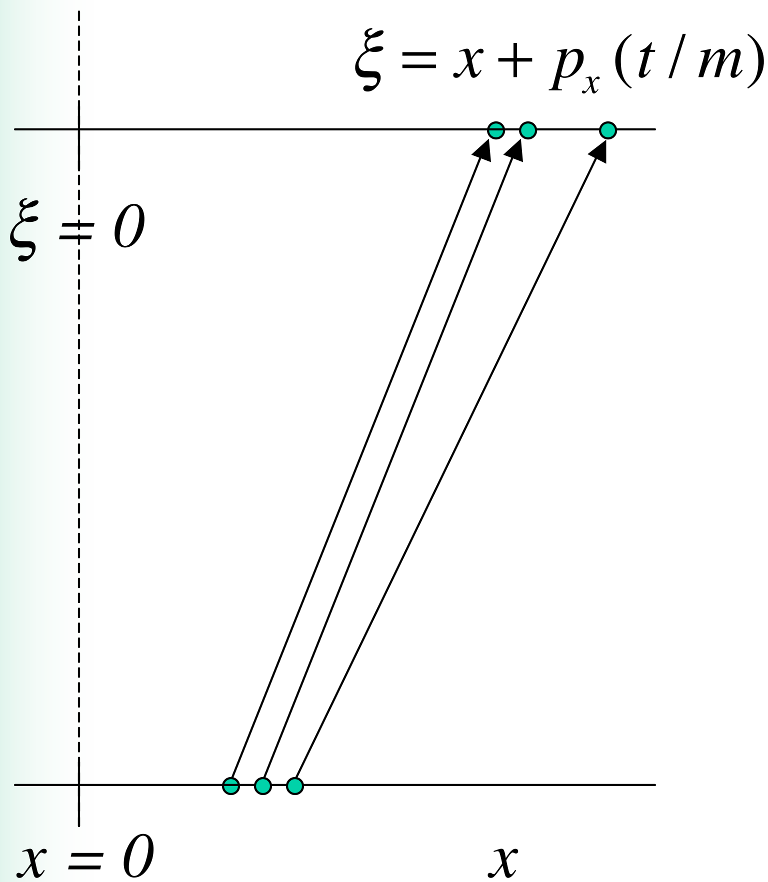
Joint Probability: $\Pr(x, p_x)$



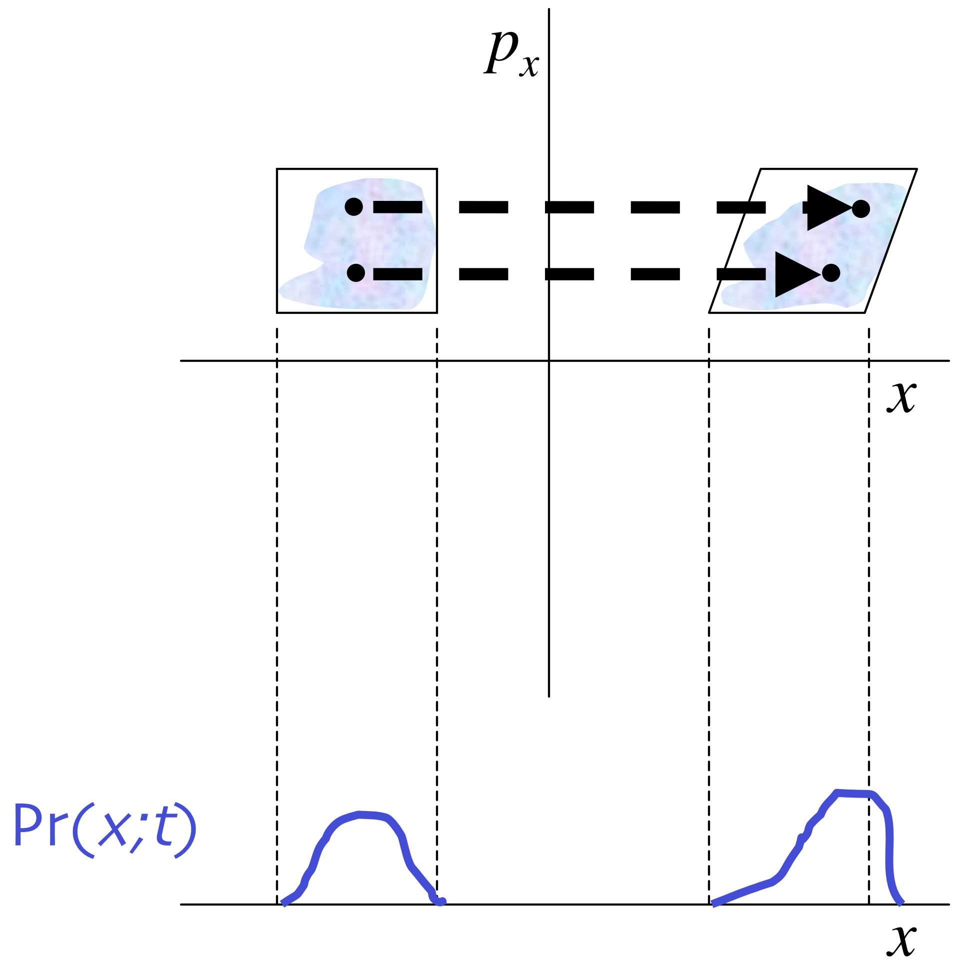
$$\xi = x + p_x (t/m)$$

$$\Pr(\xi;t) = \int \int \Pr(x, p_x)\delta(\xi - [x + p_x (t/m)])\,dx\,dp_x$$

EXPERIMENT WITH ATOMIC BEAM



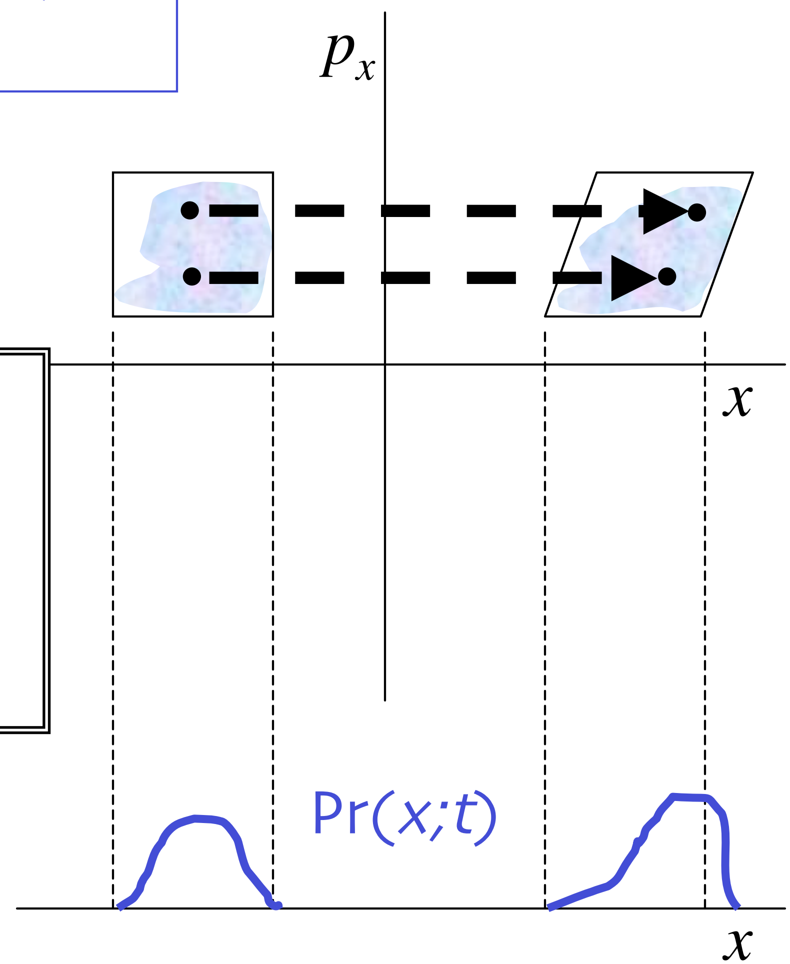
Joint Probability Distribution



INVERSION OF EXPERIMENTAL DATA

$$\Pr(\xi;t) = \int \int \underline{\Pr(x,p_x)}\,\delta(\xi - [x + p_x(t/m)])\,dx\,dp_x$$

Joint Probability Distribution

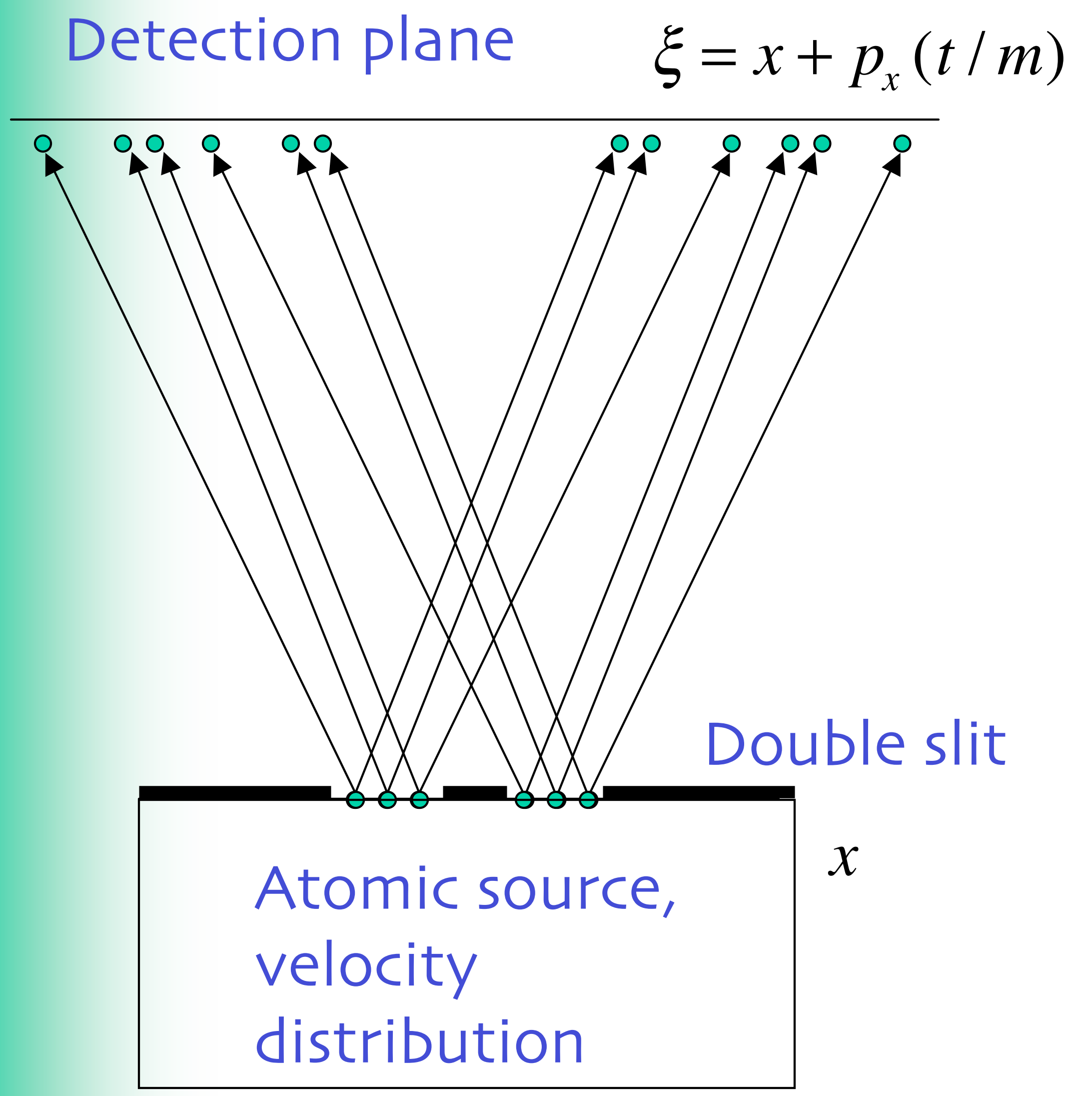


Mathematical Inversion:

$$\Pr(x,p_x) = \int \Pr(\xi;t)\,K(x,p_x;\xi;t)\,d\xi\,dt$$

EXPERIMENT WITH ATOMIC BEAM

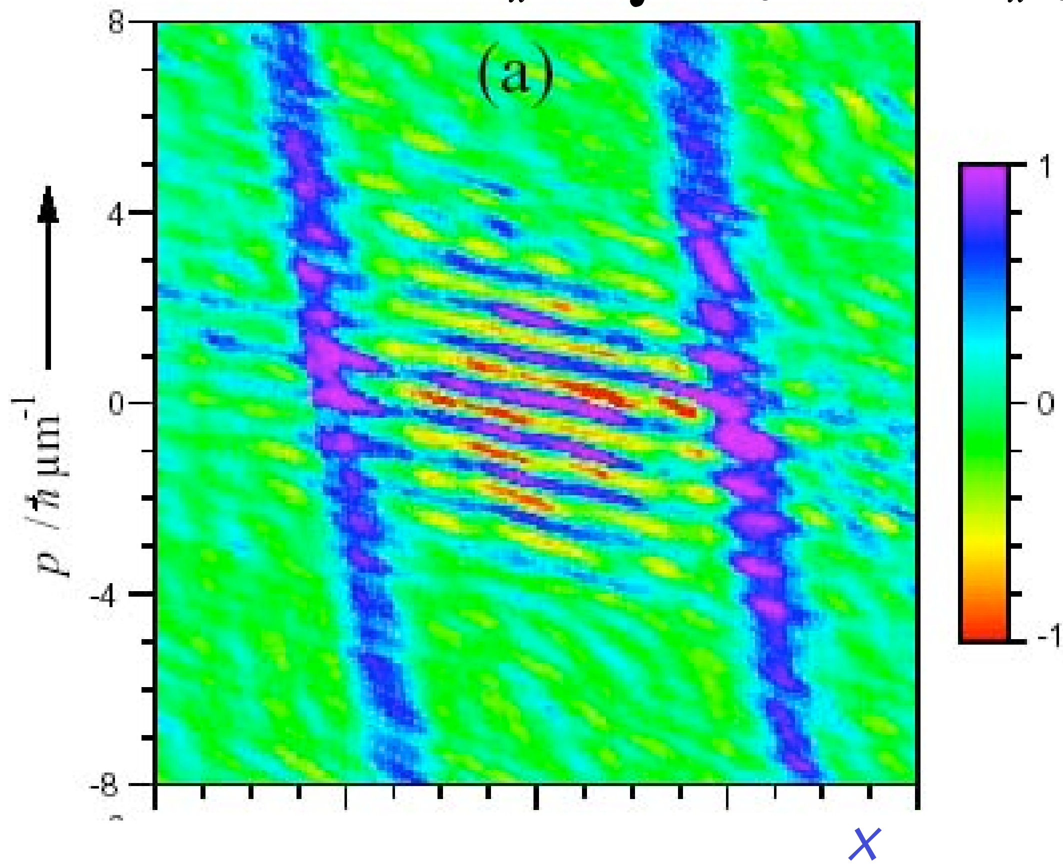
T. Pfau, C. Kurtseifer (Uni Konstanz)



Measure Pr(*x;t*) for many different flight times *t*.

Pr($x;t$) Data → Joint Probability Distribution

$$\Pr(x, p_x) = \int \Pr(\xi;t) K(x, p_x;\xi;t)\, d\xi\, dt$$



T. Pfau, C. Kurtseifer

NEGATIVE!

"Joint Probability Distribution" -> Wigner Distribution

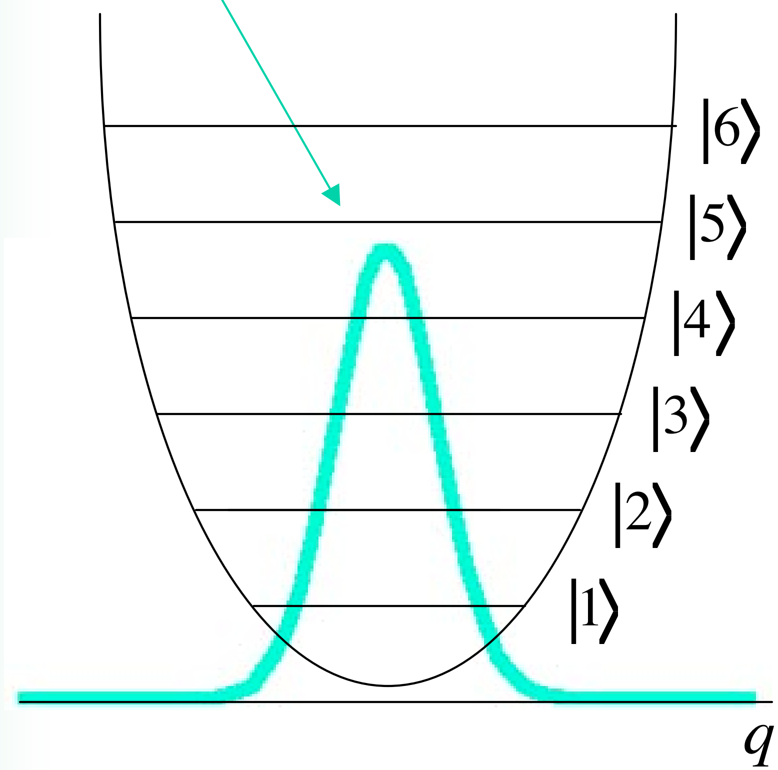
$$W(q,p) = \frac{1}{\pi} \int_{-\infty}^{\infty} \psi(q + x/2) \psi^* (q + x/2) \exp(-i p x)\, dx$$

Harmonic Oscillator Example

Deterministic inversion of statistical data to yield density matrix. Probability to observe position = q at time t :

$$\Pr(q,t) = \sum_{\psi} P(\psi)\left|\psi(t)\right|^2 = \sum_{\mu,\nu} \rho_{\mu\nu}\, \psi_{\mu}(q)\psi_{\nu}(q)\exp[i(\nu-\mu)\omega t]$$

$$\rho_{\mu\nu} = \sum_{\psi} P(\psi)\langle\mu|\psi\rangle\langle\psi|\nu\rangle$$



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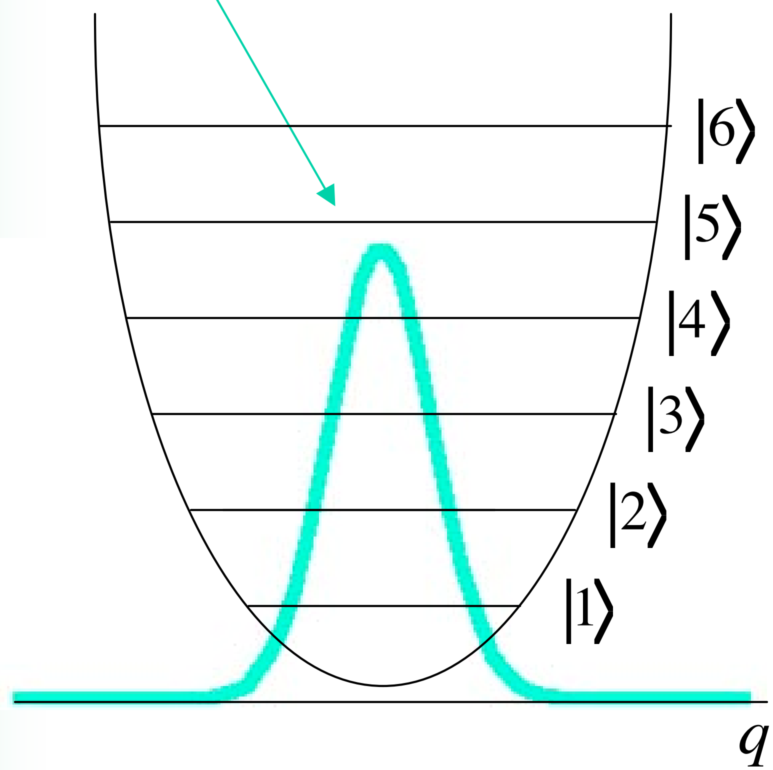
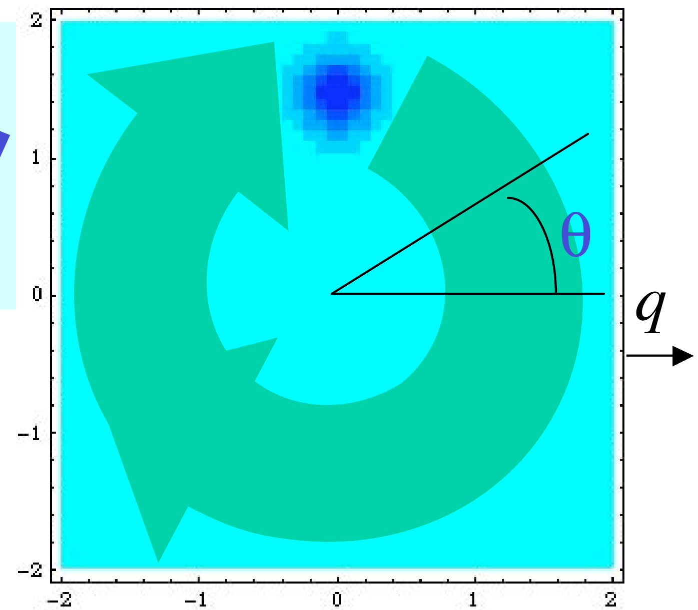


Harmonic Oscillator Example - Phase Space

$$\Pr(q,t) = \sum_{\psi} P(\psi) \left|\psi(t)\right|^2 = \sum_{\mu,\nu} \rho_{\mu\nu}\, \psi_{\mu}(q)\psi_{\nu}(q)\exp[i(\nu-\mu)\omega t]$$

Phase-Space Probability Density

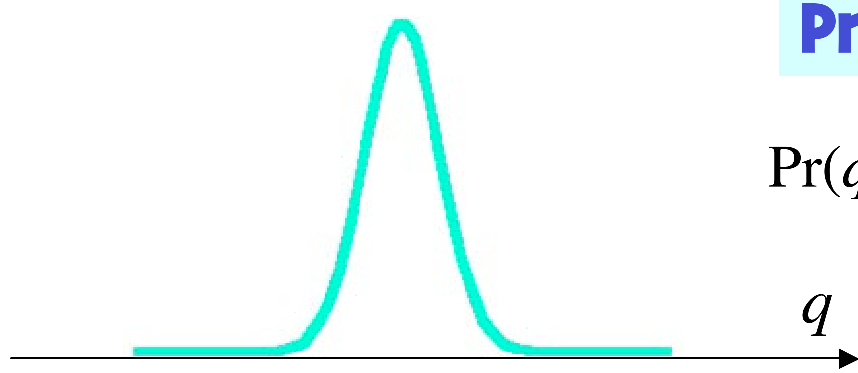
W(q,p,t)



$\theta = \omega t$ = phase angle

Projection = Probability:

$$\Pr(q,t) = \int W(q,p,t)\,dp$$



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Phase-Space Probability Density

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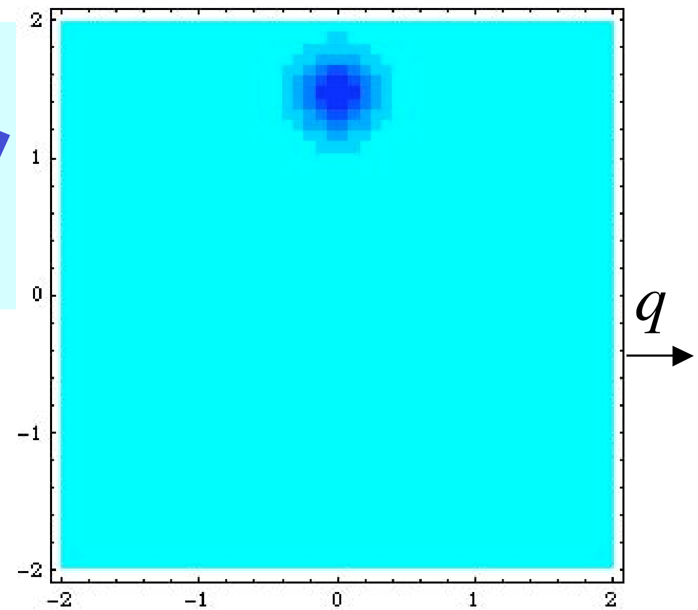

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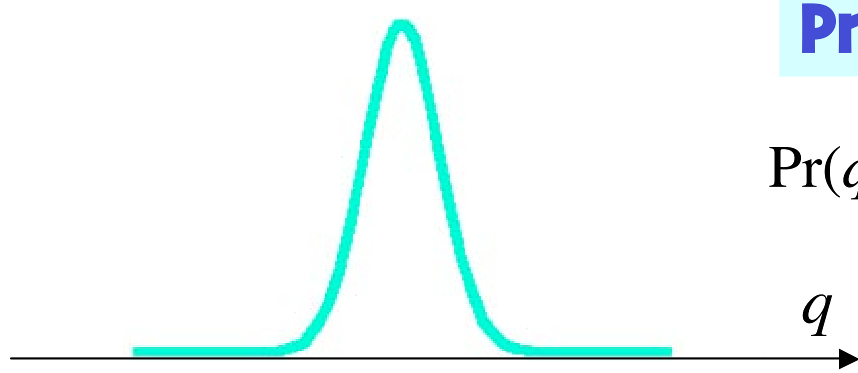


$\theta = \omega t$ = phase angle

$$\psi_n(q,t) = \psi_n(q,0) exp[-in\omega t] \leftrightarrow \psi_n(q,\theta) = \psi_n(q,0) exp[-in\theta].$$

$$\Pr(q,\theta) = \sum_{\mu,\nu} \rho_{\mu\nu} \psi_\mu(q) \psi_\nu(q) \exp[i(\nu-\mu)\theta].$$

$$\Pr(q_\theta,\theta) = \iint \delta(q_\theta - \tilde{q}_\theta(q,p)) W(q,p) dq dp.$$

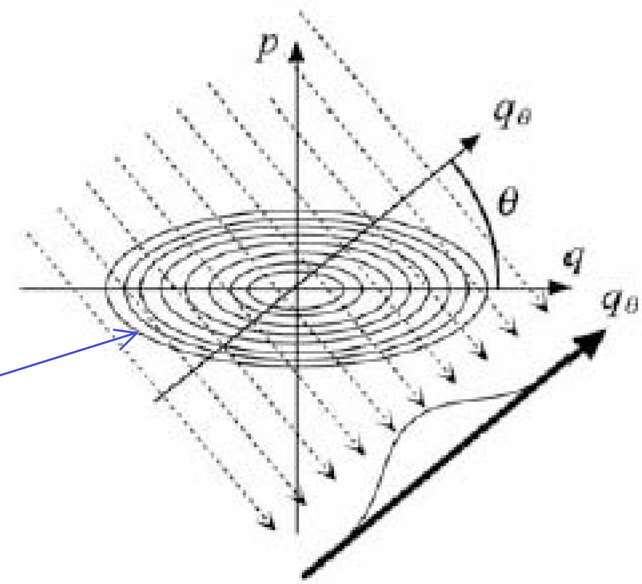
Wigner Distr.

Radon Transform

$$= q_\theta \cos\theta - p_\theta \sin\theta, \text{ and } p = q_\theta \sin\theta + p_\theta \cos\theta,$$

$$\Pr(q_\theta,\theta) = \int_{-\infty}^{\infty} W(q_\theta \cos\theta - p_\theta \sin\theta, q_\theta \sin\theta + p_\theta \cos\theta) dp_\theta.$$

Radon Transform



Wigner Distr.

For squeezed state

$$q = q_\theta \cos\theta - p_\theta \sin\theta, \text{ and } p = q_\theta \sin\theta + p_\theta \cos\theta,$$

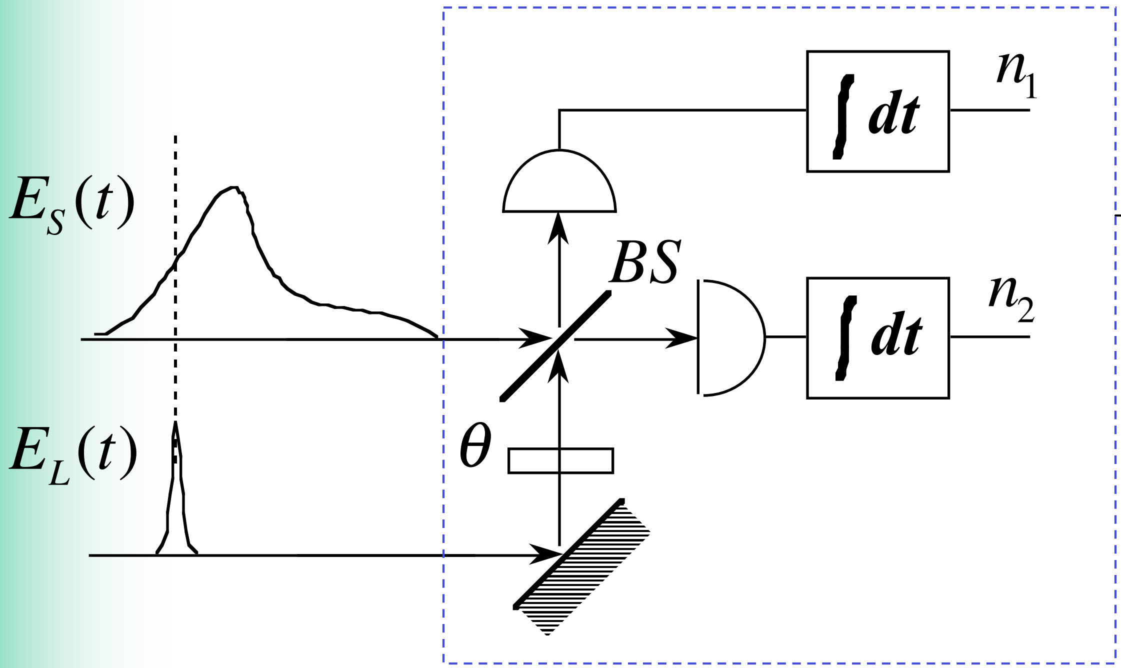
$$\Pr(q_\theta, \theta) = \int_{-\infty}^{\infty} W(q_\theta \cos\theta - p_\theta \sin\theta, q_\theta \sin\theta + p_\theta \cos\theta) dp_\theta.$$

Inverse Radon Transform

$$W(q,p) = \frac{1}{4\pi^2} \int_{-\infty}^{\infty} dq_\theta \int_{-\infty}^{\infty} d\xi\, |\xi| \int_{0}^{\pi} d\theta \Pr(q_\theta, \theta)\, e^{i\xi(q_\theta - q\cos\theta - p\sin\theta)}$$

Measuring Wigner Distributions for Optical Fields

DC-Balanced Homodyne Dection

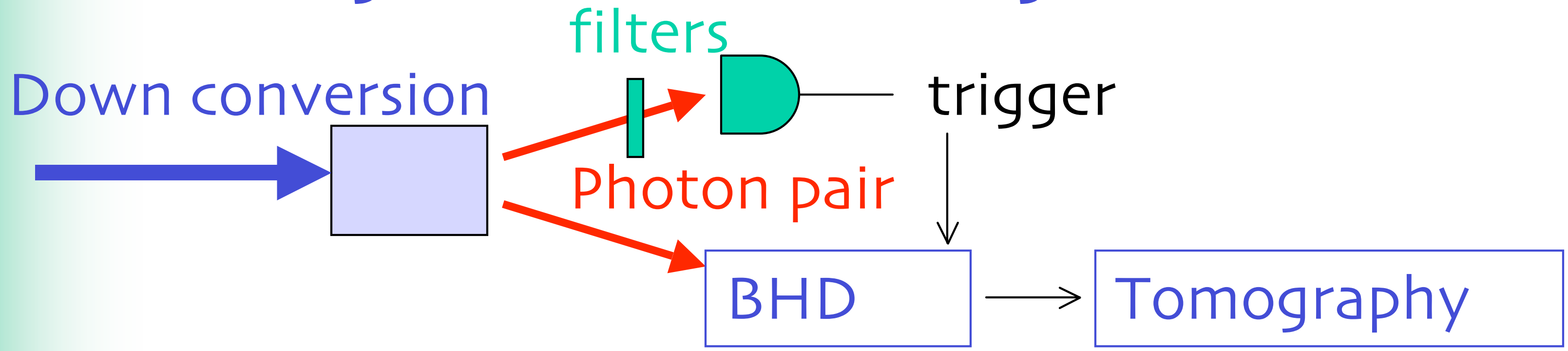


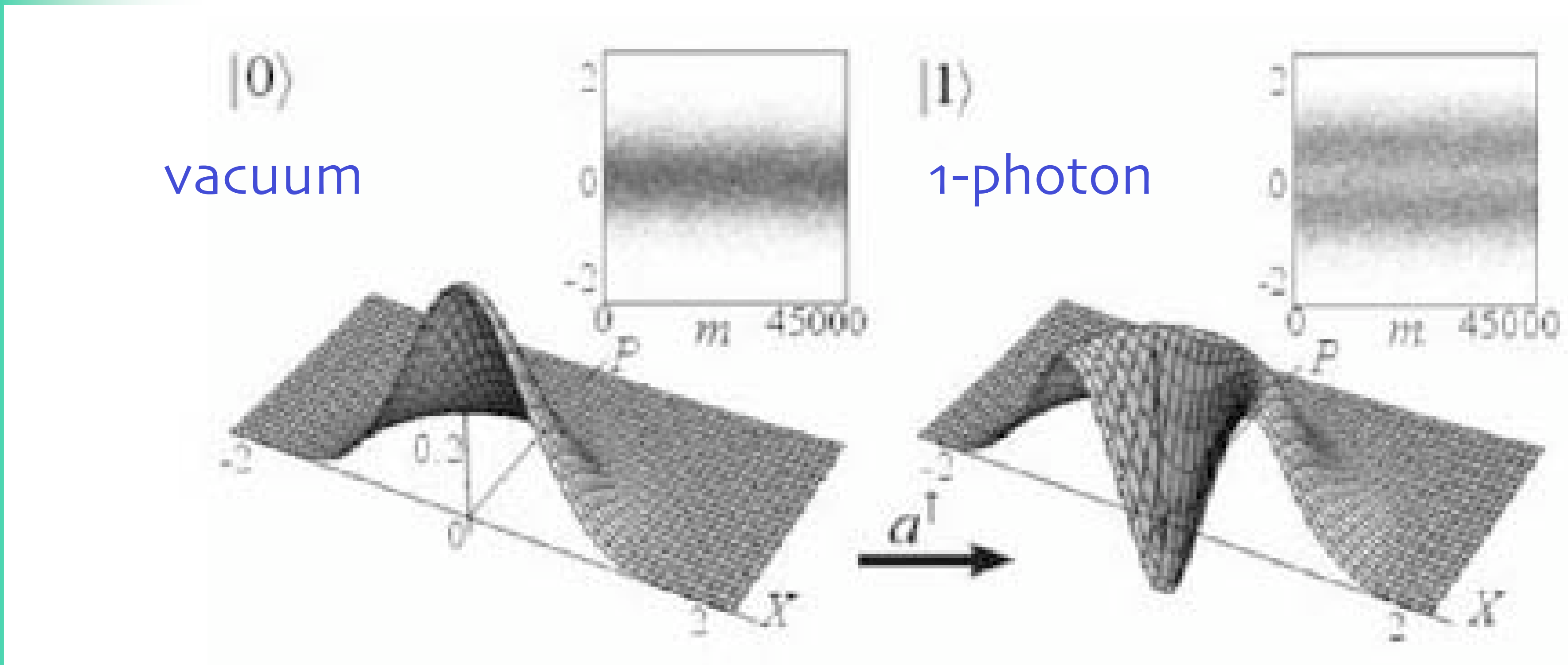
Tomography

In Phase Space

W(q,p)

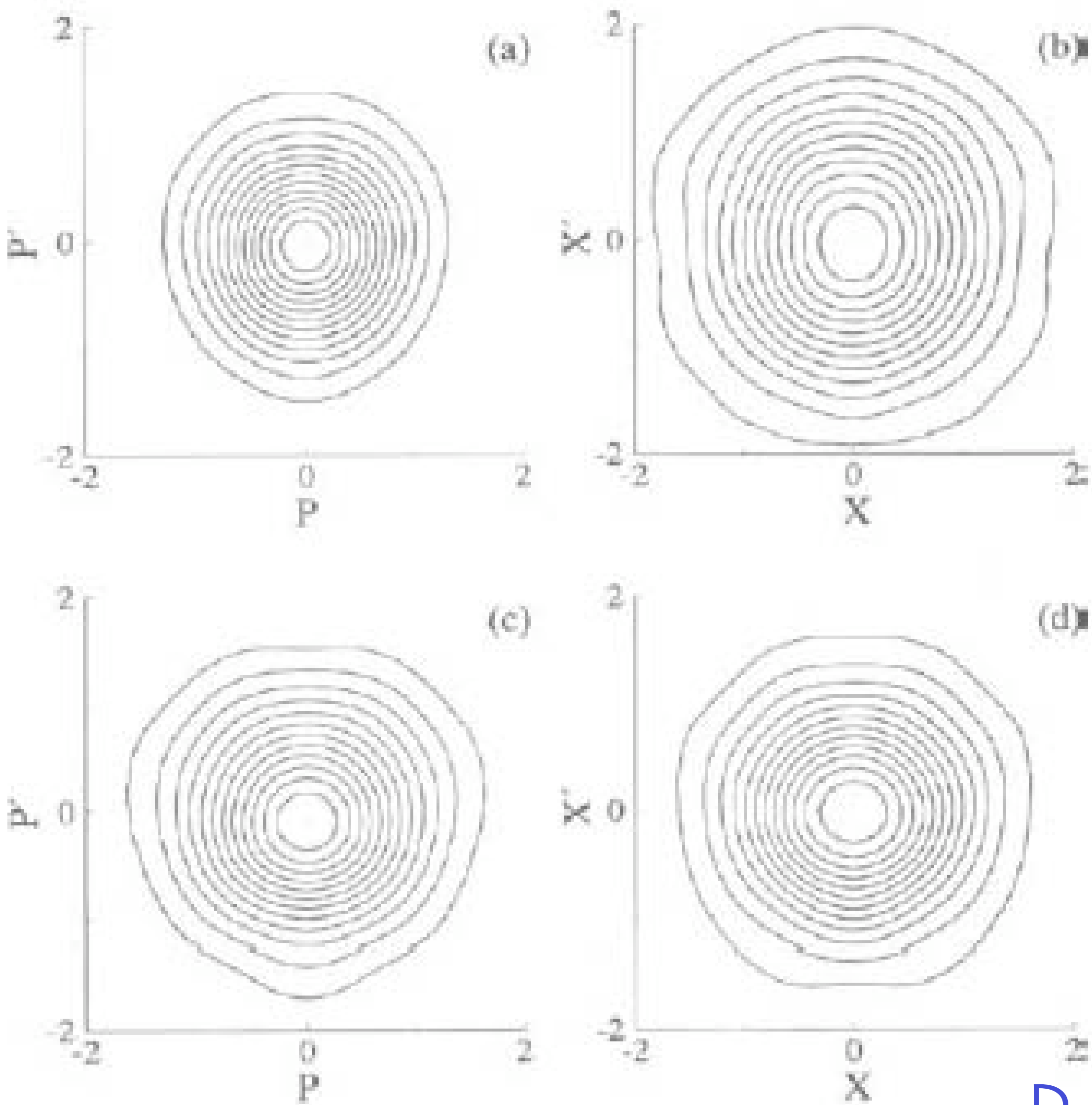
Measured Wigner distribution for Single-Photon State





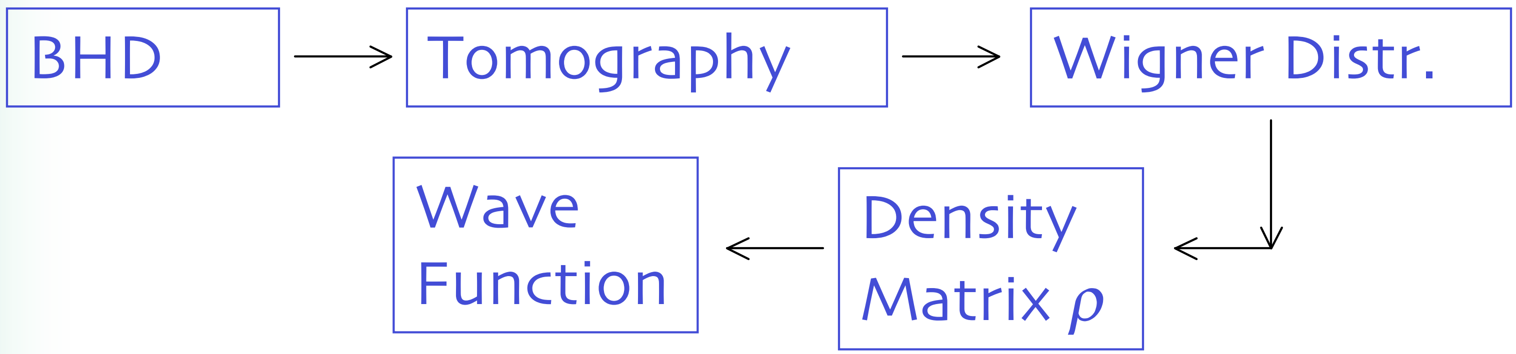
Breitenbach, Schiller, Lvovsky (Uni Konstanz)

Measured Density Matrix for Quadrature-Squeezed Light



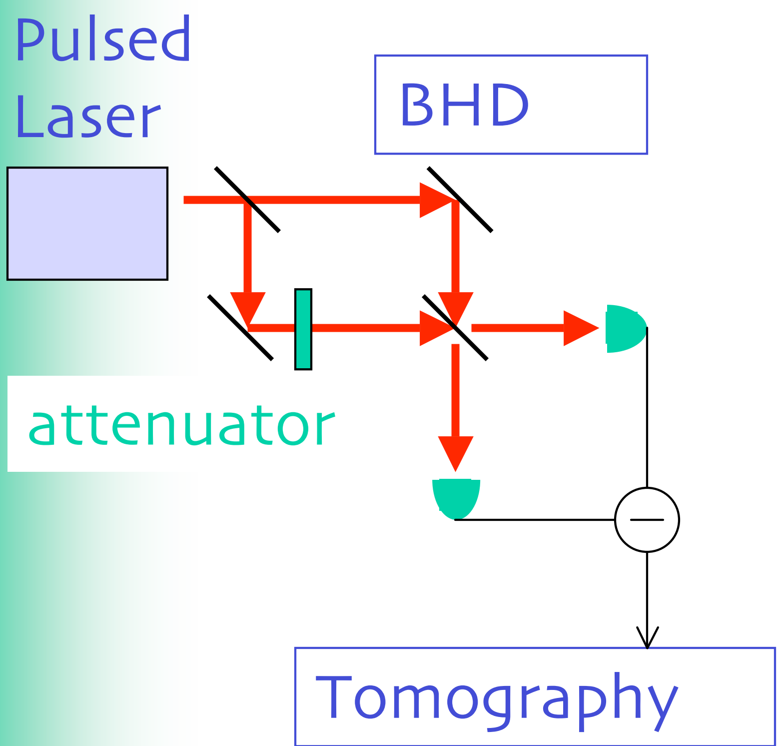
D. Smithey 1993

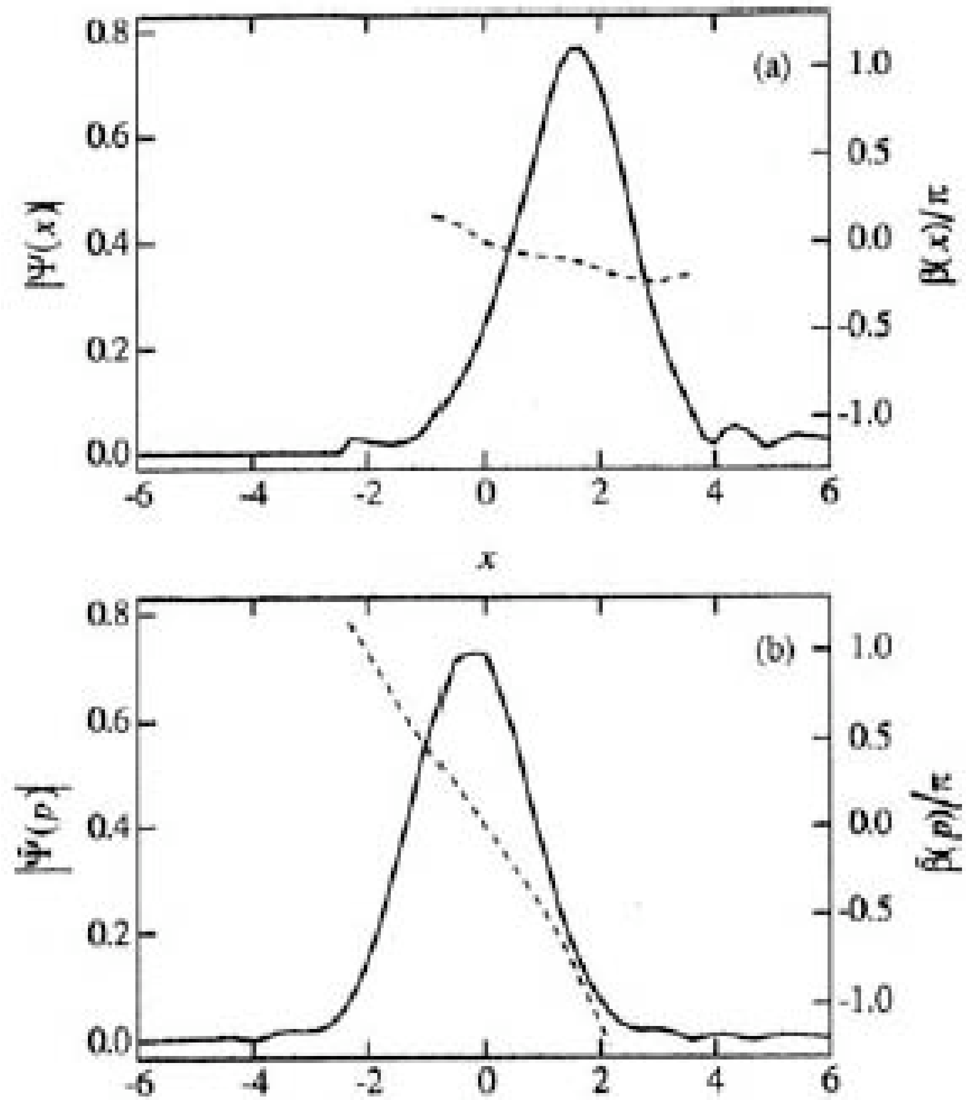
Determining a Quantum Wave Function from a Wigner Distribution



$$\psi(q) = \frac{\langle q|\hat{\rho}|q' = 0\rangle}{\sqrt{\langle q = 0|\hat{\rho}|q' = 0\rangle}} e^{i\beta_0};$$

Determined Quantum Wave Function of Coherent State from a Laser (Smithey 1993)





Quantum state tomography has entered into the standard bag of tricks in quantum optics and quantum information research.

For elementary review see:

"Measuring the quantum mechanical wave function," M. G. Raymer, Contemp. Physics **38**, 343 (1997).

For detailed reviews see:

“Experimental Quantum State Tomography of Optical Fields and Ultrafast Statistical Sampling,” (60 pages) Michael G. Raymer and Mark Beck, in *Quantum State Estimation*, eds. M. Paris and J. Rehacek, Springer Verlag (2004).

“Quantum State Tomography of Optical Continuous Variables,” with Alex Lvovsky, to appear in *Quantum Information with Continuous Variables of Atoms and Light*, eds. Nicolas Cerf, Gerd Leuchs, and Eugene Polzik (Imperial College Press, 2005)