Q: What is reality?

A: I have no idea, but experimental physics can help address this question.
PROPERTIES THE WORLD SHOULD HAVE (A. Einstein)

1. **Locality** -- all causes and effects are local.

   No influence can travel faster than the (finite) speed of light.

2. **Realism** -- physical quantities exist simultaneously, even if they cannot be measured simultaneously.

   e.g., position and momentum

Reality is defined by its “elements.”

Something is an *element of reality* if, without altering its value (or properties) in any way, one can make a correct, definite prediction about its value.
John Bell’s example (modified a la L. Hardy): Bertleman’s Socks

Prof. Bertleman is known to wear one red and one blue sock, or two blue socks, but never two red socks together.
Bertleman’s Socks

If Prof. Bertleman comes around the corner and you see a red sock, then ...

The colors of both socks are elements of Einstein’s Reality.
If each sock has two distinct properties

Each sock has a color (blue or red) and a size (LARGE or small). In every other way they are identical.

In Einstein’s world, each of these properties (color and size) should be independently measurable if they are elements of reality.

Bertleman’s laundry service delivers a thousand pairs of socks each week, bundled as pairs wrapped in paper that hides their properties. Their method for pairing up the socks is not entirely random but is unknown to Bertleman. He pulls a wrapped pair out of his hamper and unwraps it. He finds two socks, each having a particular color and size.
The first four sock pairs that he pulls out:

<table>
<thead>
<tr>
<th></th>
<th>color</th>
<th>size</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>blue</td>
<td>red</td>
</tr>
<tr>
<td>1</td>
<td>L</td>
<td>★</td>
</tr>
<tr>
<td></td>
<td>R</td>
<td>★</td>
</tr>
<tr>
<td>2</td>
<td>L</td>
<td>★</td>
</tr>
<tr>
<td></td>
<td>R</td>
<td>★</td>
</tr>
<tr>
<td>3</td>
<td>L</td>
<td>★</td>
</tr>
<tr>
<td></td>
<td>R</td>
<td>★</td>
</tr>
<tr>
<td>4</td>
<td>L</td>
<td>★</td>
</tr>
<tr>
<td></td>
<td>R</td>
<td>★</td>
</tr>
</tbody>
</table>

After many pairs are examined:

**GENERAL OBSERVATIONS:**

(left; right)

- never (blue; LARGE)
- never (LARGE; blue)
- never (red; red)

Implies that if one is LARGE, the other is red.

This would seem to imply that (LARGE; LARGE) is impossible.
GENERAL OBSERVATIONS:
never (blue; LARGE)
ever (LARGE; blue)
ever (red; red)

This would seem to imply that (LARGE; LARGE) is impossible.

ASSUME TRUE:
(LARGE; LARGE) sometimes occurs.

In those cases:
both socks must be red

This is inconsistent with never (red; red)
BUT SOMETIMES LARGE/LARGE IS OBSERVED!

Not with socks, but in analogous experiments with electrons or photons
Sock-Properties Measuring Apparatus

Blue Large

color size

B/L

R/S

color size

Red Small

hamper

no need to see socks before they are measured.
Sock-Properties Measuring Apparatus

What is wrong with this picture?
For socks, nothing, but for electrons, ...
Quantum Sock-Properties Measuring Apparatus

Color and Size could be “Incompatible Properties.”
We can measure only one of these for each sock.
We must choose what to measure.

Blue

size

measurement selector

Left Sock

color

hamper

Right Sock

color

size

measurement selector

Red

Large

size

measurement selector

Red

xxx
Produce one million sock pairs.
Choose at random the single property to measure for each sock.

<table>
<thead>
<tr>
<th>measure</th>
<th>Left Sock</th>
<th></th>
<th>Right Sock</th>
</tr>
</thead>
<tbody>
<tr>
<td>color/size</td>
<td>L ; R</td>
<td>L ; R</td>
<td>L ; R</td>
</tr>
<tr>
<td>color/size</td>
<td>Blue ; Large</td>
<td>Red ; Small</td>
<td>Red ; Large</td>
</tr>
<tr>
<td>size/color</td>
<td>Large ; Blue</td>
<td>Small ; Red</td>
<td>Small ; Blue</td>
</tr>
<tr>
<td>size/color</td>
<td>Large ; Large</td>
<td>Small ; Small</td>
<td>Small ; Large</td>
</tr>
<tr>
<td>color/color</td>
<td>Blue ; Blue</td>
<td>Red , Red</td>
<td>Red ; Blue</td>
</tr>
</tbody>
</table>

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Results: Observations cannot be explained by a model of Reality in which every “sock” has a definite color and a definite size before being measured.

Conclusion: In the Quantum World it is not true that every sock has a definite color and a definite size before being measured. It is not simply the case that definite properties exist but we just don’t know them.
Photons instead of Socks

A photon is an elementary “particle” of light. Light is an oscillating electric and magnetic force that travels in space. The direction of oscillation is called *polarization*.

Can choose axis of polarization to be Horizontal or Vertical

![Diagram showing H and V polarization](image-url)
Photons
Can alternatively choose axis of polarization to be Plus 45 degrees or Minus 45 degrees.

Measure on H / V axes

Measure on P / M axes
Measuring H/V Photon Polarization: Polarizing beam splitter

Measuring the “degree of verticalness”
Measuring P/M Photon Polarization: Polarization Rotator

Measuring the “degree of plusness”
Produce one million photon pairs.
Choose at random the polarization axis to measure for each photon.

Results: Results cannot be explained by a model of Reality in which every photon has a definite H / V polarization and a definite P / M polarization before being measured.

Conclusion: It is not true that every photon has a definite H / V polarization and a definite P / M polarization before being measured.
What state of two photons can produce such results?

Label states using Dirac’s notation. Single photons:

\[ |V\rangle_L \quad \text{V polarized (Photon on Left)} \]
\[ |H\rangle_R \quad \text{H polarized (Photon on Right)} \]

State of a pair of photons:

\[ |\Psi\rangle = |V\rangle_L |H\rangle_R \]

the state discussed by Lucien Hardy (Phys. Rev. Lett. 71, 1665, 1993)
\[ |\Psi\rangle = \frac{1}{\sqrt{3}} \left[ |V\rangle_L |H\rangle_R + |H\rangle_L |V\rangle_R - |H\rangle_L |H\rangle_R \right] \]

\[ |H\rangle = \frac{1}{\sqrt{2}} |P\rangle - \frac{1}{\sqrt{2}} |M\rangle \]

\[ |V\rangle = \frac{1}{\sqrt{2}} |P\rangle + \frac{1}{\sqrt{2}} |M\rangle \]

Measure on
H / V axes

Measure on
P / M axes
\[ |\Psi\rangle = \frac{1}{\sqrt{3}} \left[ |V\rangle_L |H\rangle_R + |H\rangle_L |V\rangle_R - |H\rangle_L |H\rangle_R \right] \]

\[ |H\rangle = \frac{1}{\sqrt{2}} |P\rangle - \frac{1}{\sqrt{2}} |M\rangle \quad ; \quad |V\rangle = \frac{1}{\sqrt{2}} |P\rangle + \frac{1}{\sqrt{2}} |M\rangle \]

\[ |\Psi\rangle = 0 \cdot |H\rangle_L |P\rangle_R - \frac{1}{\sqrt{6}} |V\rangle_L |M\rangle_R + \frac{1}{\sqrt{6}} |V\rangle_L |P\rangle_R + \sqrt{\frac{2}{3}} |H\rangle_L |M\rangle_R \]

\[ |\Psi\rangle = 0 \cdot |P\rangle_L |H\rangle_R - \frac{1}{\sqrt{6}} |M\rangle_L |V\rangle_R + \sqrt{\frac{2}{3}} |M\rangle_L |H\rangle_R + \frac{1}{\sqrt{6}} |P\rangle_L |V\rangle_R \]

\[ |\Psi\rangle = \frac{1}{\sqrt{12}} |P\rangle_L |P\rangle_R - \sqrt{\frac{3}{4}} |M\rangle_L |M\rangle_R + \frac{1}{\sqrt{12}} |M\rangle_L |P\rangle_R + \frac{1}{\sqrt{12}} |P\rangle_L |M\rangle_R \]

\[ |\Psi\rangle = -\frac{1}{\sqrt{3}} |H\rangle_L |H\rangle_R + 0 \cdot |V\rangle_L |V\rangle_R + \frac{1}{\sqrt{3}} |V\rangle_L |H\rangle_R + \frac{1}{\sqrt{3}} |H\rangle_L |V\rangle_R \]
### The Bertleman State

<table>
<thead>
<tr>
<th>measure</th>
<th>L ; R</th>
<th>L ; R</th>
<th>L ; R</th>
<th>L ; R</th>
</tr>
</thead>
<tbody>
<tr>
<td>color/size</td>
<td>Blue ; Large</td>
<td>Red ; Small</td>
<td>Red ; Large</td>
<td>Blue ; Small</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>1/6</td>
<td>1/6</td>
<td>2/3</td>
</tr>
<tr>
<td>size/color</td>
<td>Large ; Blue</td>
<td>Small ; Red</td>
<td>Small ; Blue</td>
<td>Large ; Red</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>1/6</td>
<td>2/3</td>
<td>1/6</td>
</tr>
<tr>
<td>size/size</td>
<td>Large ; Large</td>
<td>Small ; Small</td>
<td>Small ; Large</td>
<td>Large ; Small</td>
</tr>
<tr>
<td></td>
<td>1/12</td>
<td>3/4</td>
<td>1/12</td>
<td>1/12</td>
</tr>
<tr>
<td>color/color</td>
<td>Blue ; Blue</td>
<td>Red , Red</td>
<td>Red ; Blue</td>
<td>Blue ; Red</td>
</tr>
<tr>
<td></td>
<td>1/3</td>
<td>0</td>
<td>1/3</td>
<td>1/3</td>
</tr>
</tbody>
</table>
## The Hardy State

<table>
<thead>
<tr>
<th>Measure</th>
<th>$L;R$</th>
<th>$L;R$</th>
<th>$L;R$</th>
<th>$L;R$</th>
</tr>
</thead>
<tbody>
<tr>
<td>HV;PM</td>
<td>$0 \cdot</td>
<td>H\rangle_L</td>
<td>P\rangle_R$</td>
<td>$-\frac{1}{\sqrt{6}}</td>
</tr>
<tr>
<td>PM;HV</td>
<td>$0 \cdot</td>
<td>P\rangle_L</td>
<td>H\rangle_R$</td>
<td>$-\frac{1}{\sqrt{6}}</td>
</tr>
<tr>
<td>PM;PM</td>
<td>$\frac{1}{\sqrt{12}}</td>
<td>P\rangle_L</td>
<td>P\rangle_R$</td>
<td>$-\sqrt{\frac{3}{4}}</td>
</tr>
<tr>
<td>HV;HV</td>
<td>$-\frac{1}{\sqrt{3}}</td>
<td>H\rangle_L</td>
<td>H\rangle_R$</td>
<td>$0 \cdot</td>
</tr>
</tbody>
</table>
There appear to be certain quantities in nature that are not *Elements of Reality* in the sense that Einstein hoped for.