

Quantum efficiency of single-photon sources in the cavity-QED strong-coupling regime

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Abstract: We calculate the integrated-pulse quantum efficiency of single-photon sources in the cavity quantum electrodynamics (QED) strong-coupling regime. An analytical expression for the quantum efficiency is obtained in the Weisskopf-Wigner approximation. Optimal conditions for a high quantum efficiency and a temporally localized photon emission rate are examined. We show the condition under which the earlier result of Law and Kimble [J. Mod. Opt. 44, 2067 (1997)] can be used as the first approximation to our result.

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1. Introduction

Various implementations of single-photon sources (SPS) based on atom-like emitters have been reported based on different systems in the last three decades, such as calcium atoms [2], single ions in traps [3], single molecules [4], a color center in diamond[5], and semiconductor nanocrystals [6] or quantum dots (QD) [7, 8]. The need for efficient single-photon sources, however, is still a major challenge in the context of quantum information processing [9, 10]. In order to efficiently produce single photons on demand, the single quantum emitter is coupled to a resonant high-finesse optical cavity. A cavity can channel the spontaneously emitted photons into a well-defined spatial mode and in a desired direction to improve the collection efficiency, and can alter the spectral width of the emission. It can also provide an environment where dissipative mechanisms are overcome so that a pure-quantum-state emission takes place. A major question is what is the quantum efficiency (QE) of the emission from such systems.

Depending on the ratios of the coherent interaction rate g_0 between the quantum-emitter and cavity, to the intracavity field decay rate 2κ , and to the emitter population decay rate 2γ , we can distinguish two regimes of coupling between the emitter and the cavity: strong coupling for $g_0 > \kappa, \gamma$ and weak coupling for $g_0 < \kappa, \gamma$. The realizations of cavity-QED strong coupling in the atom-cavity [11] and QD-cavity systems [12-14] allow researchers to deterministically generate single photons [15, 16]. Single-atom lasers in the strong-coupling regime have also been studied [17]. While not in the strong-coupling regime, Santori et al. [18] showed the ability to produce largely indistinguishable photons by a semiconductor QD in a microcavity using a large Purcell effect [19]. The QE (η_q) of SPS, which is intrinsic to the composite quantum system, can be different in these two regimes because the dynamics of the composite system is different. The overall efficiency of SPS will also depend on the excitation efficiency, collection efficiency and detection efficiency, which are not intrinsic to the composite quantum system; however, they can be greatly affected by the energy structure of the quantum emitter and the geometry of the cavity. Qualitative discussions of different efficiencies based on a particular system in the Purcell (bad-cavity) regime have been reported in the literature elsewhere [20].

In this paper we calculate the integrated-pulse QE of SPS in the cavity QED strong-coupling regime based on the solutions of the probability amplitudes in the Weisskopf-Wigner approximation (WWA) [21]. We find that the QE equals

$$\eta_q = \left[g_0^2 / (g_0^2 + \kappa\gamma) \right] \cdot \left[\kappa / (\kappa + \gamma) \right]. \quad (1)$$

We show the condition under which earlier result associated with Law and Kimble et al. in [1] can be used as the first approximation to this more complete result. We also establish the

connection between our analytical results and the qualitative discussions of Pelton et al. in [20].

2. Probability-amplitude method in the Weisskopf-Wigner approximation

Consider the interaction of a quantized radiation field with a two-level emitter located at an antinode of the field in an optical microcavity, as in Figure 1. M_1 is a perfect 100%-reflecting mirror and M_2 is a partially transparent one, from which a sequence of single photons-on-demand emerges.

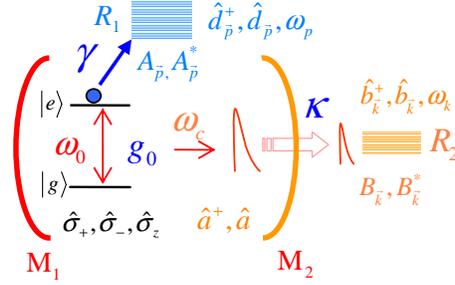


Fig. 1. Schematic description of a lossy two-level emitter interacting with a single mode in a leaky optical cavity. g_0 is the coupling constant between the emitter and the cavity field.

$A_{\bar{p}}$, $A_{\bar{p}}^*$ and $B_{\bar{k}}$, $B_{\bar{k}}^*$ are the coupling constants between the emitter, a single photon and their respective reservoir (R_1 , R_2) fields.

The interaction Hamiltonian \hat{H}_I in the interaction picture for this system in the dipole approximation and rotating-wave approximation is [22],

$$\hat{H}_I(t) = \hbar g_0 (\hat{\sigma}_+ \hat{a} e^{i\Delta t} + h.c.) + \hbar \sum_{\bar{p}} (A_{\bar{p}}^* \hat{\sigma}_- \hat{d}_{\bar{p}}^+ e^{i\delta_{\bar{p}} t} + h.c.) + \hbar \sum_{\bar{k}} (B_{\bar{k}}^* \hat{a} \hat{b}_{\bar{k}}^+ e^{i\delta_{\bar{k}} t} + h.c.) \quad (2)$$

where $\Delta = \omega_0 - \omega_c$, $\delta_{\bar{p}} = \omega_p - \omega_0$, $\delta_{\bar{k}} = \omega_k - \omega_c$ are the detunings of the emitter-cavity, emitter-reservoir, and cavity-reservoir. \hat{a} and \hat{a}^+ are the annihilation and creation operators for the single cavity mode under consideration, while $\hat{\sigma}_z$ and $\hat{\sigma}_{\pm}$ are the Pauli operators for the emitter population inversion, raising, and lowering, respectively.

At arbitrary time t , the state vector can be written as

$$|\psi(t)\rangle = E(t) |e, 0\rangle |0\rangle_{R_1} |0\rangle_{R_2} + C(t) |g, 1\rangle |0\rangle_{R_1} |0\rangle_{R_2} + \sum_{\bar{p}} S_{\bar{p}}(t) |g, 0\rangle |1_{\bar{p}}\rangle_{R_1} |0\rangle_{R_2} + \sum_{\bar{k}} O_{\bar{k}}(t) |g, 0\rangle |0\rangle_{R_1} |1_{\bar{k}}\rangle_{R_2} \quad (3)$$

where $|m, n\rangle$ ($m = e, g$, $n = 0, 1$) denotes the emitter state $|m\rangle$ (excited state $|e\rangle$, ground state $|g\rangle$) with n photons in the cavity. $|j_{\bar{p}}\rangle_{R_1} |l_{\bar{k}}\rangle_{R_2}$ ($j, l = 0, 1$) corresponds to j photons in the \bar{p} mode (other than the privileged cavity mode) of the emitter reservoir R_1 and l photons in a single-mode (\bar{k}) traveling wave of the one-dimensional photon reservoir R_2 (output beam). $E(t)$, $C(t)$, $S_{\bar{p}}(t)$ and $O_{\bar{k}}(t)$ are complex probability amplitudes.

The equations of motion for the probability amplitudes are obtained by substituting $|\psi(t)\rangle$ and $\hat{H}_I(t)$ into the Schrödinger equation and then projecting the resulting equations onto different states respectively. In the WWA [21, 22], we obtain

$$\dot{E}(t) = -ig_0 \exp(i\Delta t)C(t) - \gamma E(t), \quad \dot{C}(t) = -ig_0 \exp(-i\Delta t)E(t) - \kappa C(t) \quad (4)$$

$$S_{\bar{p}}(t) = -iA_{\bar{p}}^* \int_0^t dt'' \exp(i\delta_p t'') E(t''), \quad O_{\bar{k}}(t) = -iB_{\bar{k}}^* \int_0^t dt' \exp(i\delta_k t') C(t') \quad (5)$$

where γ and κ are one-half the radiative decay rates of the emitter population (other than the privileged cavity mode) and the intracavity field, respectively.

Consider the case that the emitter and cavity are at resonance, $\Delta = \omega_0 - \omega_c = 0$. By using the initial conditions that at time $t_0 = 0$ the quantum emitter is prepared in its excited state $E(0) = 1$, $C(0) = 0$, we obtain the solutions to Eq. (4),

$$E(t) = \exp[-(\mathbf{K}/2)t] \cdot \left[\cos(gt) + \frac{\Gamma}{2g} \sin(gt) \right] \quad (6)$$

$$C(t) = \exp[-(\mathbf{K}/2)t] \cdot \left[-\frac{ig_0}{g} \sin(gt) \right] \quad (7)$$

where $\mathbf{K} = \kappa + \gamma$, $\Gamma = \kappa - \gamma$, and $g \equiv [g_0^2 - (\Gamma/2)^2]^{1/2}$ is the generalized vacuum Rabi frequency. $S_{\bar{p}}(t)$ and $O_{\bar{k}}(t)$ can be obtained by carrying out the integrations in Eq. (5).

3. Quantum efficiency of SPS in the cavity QED strong-coupling regime

A single photon will certainly be emitted from the excited emitter, but it might not be coupled into a single-mode traveling wavepacket because it can also spontaneously decay to the emitter reservoir. Define the emission probability $P_o(t)$ to be the probability of finding a single photon in the output mode of the cavity between the initial time $t_0 = 0$ and a later time t . This equals

$$P_o(t) = 2\kappa \int_0^t dt' |C(t')|^2 = \eta_q \left\{ 1 - \exp(-\mathbf{K}t) \left[1 + \frac{\mathbf{K}^2}{2g^2} \sin^2(gt) + \frac{\mathbf{K}}{2g} \sin(2gt) \right] \right\} \quad (8)$$

where η_q is given in Eq. (1), by the single-photon emission probability $P_o(t)$ in the sufficiently long-time limit $t \gg \mathbf{K}^{-1}$. It may be decomposed as $\eta_q \equiv \eta_c \cdot \eta_{extr}$, with

$$\eta_c = \frac{g_0^2}{g_0^2 + \kappa\gamma} \equiv \frac{2C_0}{2C_0 + 1}, \quad \eta_{extr} = \frac{\kappa}{\kappa + \gamma} \quad (9)$$

where $C_0 \equiv g_0^2/2\gamma\kappa$ is the cooperativity parameter per emitter [23].

We define η_q as the quantum efficiency of SPS in the cavity-QED strong-coupling regime, which can be viewed as the product of the coupling efficiency (η_c) of the emitter to

the cavity mode and the extraction efficiency (η_{extr}) of the single photon into a single-mode traveling wavepacket. The coupling efficiency characterizes how strong the emitter is coupled to the privileged cavity mode. The extraction efficiency determines how large the fraction of light is coupled to a single wave-packet, outward-traveling-wave mode. We emphasize that the cavity decay is not considered as a loss, but rather as a coherent out-coupling, because our goal is to extract single photons from the cavity.

The photon emission rate $n(t)$, defined as the time derivative of the emission probability, gives the rate of a single photon emerging from the cavity mirror M_2 and is

$$n(t) \equiv \frac{dP_o(t)}{dt} = 2\kappa \frac{g_0^2}{g^2} \exp(-\kappa t) \sin^2(gt) \quad (10)$$

We expect the shape of $n(t)$ to be sufficiently narrow as to define a well-localized photon wavepacket and a well-specified time interval between successively emitted single photons.

From Eq. (9), we can see that the larger the ratios $g_0^2/\kappa\gamma$ and κ/γ , the higher the coupling efficiency and the extraction efficiency, respectively. For a given quantum emitter, with no pure dephasing processes, the dipole dephasing rate is limited by its population decay rate. However, we can design a cavity with a proper cavity decay rate κ to optimize the QE of SPS and the shape of the photon-emission rate. Figure 2 shows plots of the emission probabilities and the emission rates for three cavity regimes where we varied the cavity decay rate κ , given realistic parameters $(g_0, \gamma)/2\pi = (50, 1)\text{GHz}$ in each case.

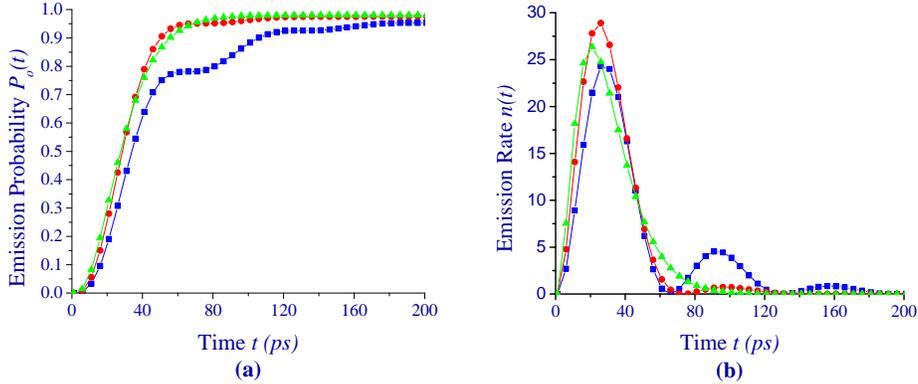


Fig. 2. Plots for the time dependence of (a) the emission probabilities of single photons $P_o(t)$, and (b) the emission rates $n(t)$, in three different cavity regimes: optimal cavity regime for $\kappa = g_0^2/\kappa \gg \gamma$, good cavity regime for $g_0^2/\kappa > \kappa \gg \gamma$, and bad cavity regime for $\kappa > g_0^2/\kappa \gg \gamma$, (red dot, blue square and green triangle, respectively) with $\kappa/2\pi = (50, 20, 100)\text{GHz}$, respectively.

We find that the optimal condition for a high QE and a temporally narrow emission rate, by optimizing the three parameters in Eq. (1), is $\kappa = g_0^2/\kappa \gg \gamma$, as shown by the red dotted curves in Figure 2. The QE is 96%, predicted by Eq. (1) in this example. The photon emission rate is well localized on the time axis. The width of $n(t)$ is about 32ps .

4. Discussion and conclusion

An earlier result obtained in the bad-cavity limit by Law and Kimble is given by [1],

$$P(t) \approx \frac{2C_1}{2C_1 + 1} \quad (11)$$

where $C_1 \equiv g_0^2/\kappa\gamma_1$ is the single-atom cooperativity parameter. Note that the γ_1 in definition (11) is the full width of the atomic absorption line. The cooperativity parameter defined in the present context is $C_0 \equiv g_0^2/2\gamma\kappa$ because here γ is the half width, so these definitions are the same. Comparing our analytical result with that given by Eq. (11), we see that Eq. (11) is valid in the limit that spontaneous atomic decay is negligible, as treated in [1], or equivalently the extraction efficiency η_{ext} is unity. This is not necessary for strong coupling and is also not implied by the strong-coupling condition. However, for deterministic production of single photons on demand, we not only require that the coupling of the emitter to the single cavity mode is far stronger than its coupling to all other modes ($g_0^2/\kappa \gg \gamma$), but also that there needs to be almost no dephasing of the emitter during the emission process ($\gamma^{-1} \gg \kappa^{-1}$). This keeps the emission process deterministic and hence guarantees that the consecutively emitted photons are indistinguishable.

The Purcell factor, widely referred to in the weak-coupling regime, is given in [19] by $F_p = (3\lambda^3/4\pi^2) \cdot (Q/V)$, which can be shown to be equal to $F_p = g_0^2/\kappa\gamma_0 \equiv 2C_0 \cdot f$, where γ_0 is one half the free-space spontaneous decay rate and $f \equiv \gamma/\gamma_0$ is the fraction of the spontaneous emission to the modes other than the privileged cavity mode. So our result for QE can also be written as

$$\eta_q = \frac{F_p}{F_p + f} \cdot \frac{\kappa}{\kappa + \gamma} = \beta \cdot \frac{\kappa}{\kappa + \gamma} \quad (12)$$

where $\beta \equiv F_p/(F_p + f)$ is called the spontaneous-emission coupling factor, the fraction of the light emitted by an emitter that is coupled into one particular mode [24, 25]. In reference [20], the authors discussed the coupling factor and the extraction efficiency in terms of the quality factor of the mode. The result Eq. (12) quantifies this discussion.

To conclude, our result for the QE of SPS in the cavity-QED strong-coupling regime is more general than earlier results in [1, 20]. It can be used to estimate the QE of single photons deterministically generated in the cavity output in the cavity-QED strong-coupling regime, instead of using the Mandel-Q parameter [15]. One can improve the QE and performance of the SPS by optimizing the three parameters in the analytical result Eq. (1). The QE is crucial for a practical use of SPS, for example, a high efficiency is required for implementing the linear-optics quantum computation schemes proposed by Knill et al. in [10]; while a low efficiency will severely limit the practical application of SPS in quantum key distribution, as shown in [26].

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