

# Spectral Broadening of Stochastic Light Intensity-Smoothed by a Saturated Semiconductor Optical Amplifier

Michael J. Munroe, John Cooper, and Michael G. Raymer

**Abstract**—We present calculations of the intensity smoothing of stochastic light by a semiconductor optical amplifier (SOA). We predict spectral changes of the light that are due to amplitude-to-phase coupling in the gain medium. The intensity smoothing of noisy optical signals with an SOA carries a penalty of spectral broadening (increased phase noise) that increases with increasing alpha parameter (linewidth enhancement factor). The spectral broadening also increases with increasing strength of the intensity fluctuations being smoothed.

**Index Terms**—Amplifier noise, optical amplifiers, optical propagation in nonlinear media, optical saturation, optical signal processing, quantum-well devices, semiconductor optical amplifiers, spectroscopy.

THE COMBINATION of gain saturation and large nonsaturable, distributed loss in a dye optical amplifier has been shown to stabilize the amplified pulse energy [1] and smooth the intensity fluctuations of noisy pulses [2], [3]. Pulses of chaotic (thermal) light were intensity-smoothed while the spectral characteristics remained relatively unchanged, as reported in [3], producing intensity-stabilized light with chaotic phase statistics. Advances in semiconductor optical amplifier (SOA) technology make possible higher gains and longer lengths than in the past [4], [5] and make SOA's an attractive alternative to dye amplifiers in performing intensity smoothing of optical pulses for signal processing applications. Using SOA's would be advantageous in signal processing applications due to their small size, high efficiency, and high optical gain. Recently, reduction in the signal fluctuations induced by crosstalk light in a wavelength division multiplexed (WDM) system has been demonstrated by passing the signal through a highly saturated, single-pass SOA [6]. An improvement in bit-error rate was reported in the detection of the amplified signal. Similarly, a reduction in intensity noise over a range of 10 dB in input intensity was observed in a double-pass semiconductor amplifier system for an input noise frequency range of 1.5 GHz [7].

However, unlike in dye amplifiers, the amplitude-to-phase coupling (alpha parameter or linewidth enhancement factor) [8], [9] present in SOA's may cause the spectrum of the intensity-smoothed light to differ from that of the input light. The amplitude-to-phase coupling present in SOA's may produce linewidth broadening related to that observed in semiconductor lasers [8], [9] and may limit the performance of these amplifiers in linewidth-sensitive systems such as WDM networks. While the intensity-smoothing characteristics of an SOA should not differ significantly from that of a dye laser amplifier and should be well modeled by past theoretical calculations [3], the spectral characteristics of the intensity-smoothed light have not yet been investigated.

This paper presents calculations of the spectral characteristics of the intensity-smoothed light from an SOA with stochastic (noisy) input light. Light with chaotic (thermal) statistics or Gaussian-amplitude statistics is modeled as an input to the smoothing amplifier. These inputs are used because they are well understood, represent an approximate model for real signals [10], [11], and allow comparison between the two results and the past work reported in [3]. Furthermore, these two models of fluctuating light represent different degrees of phase and intensity noise; chaotic fluctuating light has a mix of phase and intensity noise while Gaussian-amplitude fluctuating light has a constant phase and Gaussian amplitude fluctuations. The output optical spectrum and fringe visibility in the interference between the input and intensity-smoothed output light are presented for both input light statistics at several different values of amplitude-to-phase coupling ( $\alpha$ ). We find that the presence of amplitude-to-phase coupling results in the broadening of the optical spectrum of the output light as compared to the spectrum of the input light, and this broadening increases with increasing alpha parameter and increasing intensity noise.

We have theoretically modeled the SOA system using the standard partial differential equations [12] for the slowly varying amplitude of the electric field  $A(t, z)$  and the carrier density  $N(t, z)$  in the reference frame of the traveling pulse

$$\partial_z A(t, z) = -\beta A(t, z) + \frac{g_0}{2}(1 - i\alpha)(N(t, z) - N_0)A(t, z) \quad (1)$$

and

$$\partial_t N(t, z) = \frac{I_p}{qV} - \frac{N(t, z)}{\tau_s} - \frac{g_0}{\hbar\omega_0}(N(t, z) - N_0)|A(t, z)|^2 \quad (2)$$

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where  $\beta$  is the nonsaturable, distributed loss,  $\alpha$  is the amplitude-to-phase coupling constant,  $g_0$  is the unsaturated gain coefficient,  $N_0$  is the transparency carrier density,  $I_p$  is the injection current,  $q$  is the electron charge,  $V$  is the volume of the amplifier gain region,  $1/\tau_s$  is the spontaneous emission rate,  $\hbar$  is Planck's constant, and  $\omega_0$  is the center frequency of the gain profile, which is assumed to correspond to the center frequency of the input light. Equations (1) and (2) assume a linear gain model and neglect spontaneous emission noise.

In the case that the stimulated emission rate in (2) is much larger than any other relaxation rate in the system, we can solve for  $N(t, z)$  in quasi-steady state (as in [3]), resulting in

$$N(t, z) = N_0 + \frac{I_p}{qV} \frac{\hbar\omega_0}{g_0 I(t, z)} \quad (3)$$

where  $I(t, z) = |A(t, z)|^2$  is the intensity. Substituting (3) into (1), we can solve for the intensity to give

$$I(t, z) = I_T + (I(t, 0) - I_T) \exp(-2\beta z) \quad (4)$$

where  $I_T = I_p \hbar\omega_0 / 2qV\beta$ . Equation (4) shows that the output intensity will equal the transparency intensity  $I_T$  (intensity that results in zero net gain) for a sufficiently long amplifier, being dependent only on drive current and not input intensity.

This result is the essence of intensity smoothing: an input field having intensity fluctuations slower than the stimulated emission rate will evolve into a field having virtually no intensity fluctuations. This also confirms the intuitive conjecture that the amplitude-to-phase coupling does not affect the intensity smoothing of the amplifier.

Next we substitute (3) into (1), using (4) in the denominator of (3). This leads to a differential equation for  $A(t, z)$ , which can be solved to yield the field

$$A(t, L) = A(t, 0) \sqrt{f(L)} \exp\left(-\beta L - i\frac{\alpha}{2} \ln |f(L)|\right) \quad (5)$$

at the output of the amplifier, with length  $L$ , where

$$f(L) = \frac{I(t, 0) + I_T [\exp(2\beta L) - 1]}{I(t, 0)}. \quad (6)$$

The limit of complete intensity smoothing (an ideal smoothing amplifier) corresponds to the case  $\beta L \gg 1$ , in which case we can approximate (5) as

$$A(t, L) = A(t, 0) \sqrt{\frac{I_T}{I(t, 0)}} \cdot \exp\left\{-i\frac{\alpha}{2} \left[2\beta L + \ln\left(\frac{I_T}{I(t, 0)}\right)\right]\right\}. \quad (7)$$

In order to calculate the optical spectrum of the intensity-smoothed output light, we first derive an expression for the field correlation function  $C(\tau) = \langle A(t + \tau, L) A^*(t, L) \rangle$  as a

function of the input field  $A(t, 0)$ , where brackets indicate an ensemble average over input-field statistics. Using (7), we find

$$C(\tau) = I_T \left\langle \exp\left\{-i[\varphi(\tau, 0) - \varphi(0, 0)] + i\frac{\alpha}{2} \ln \left[\frac{I(\tau, 0)}{I(0, 0)}\right]\right\}\right\rangle \quad (8)$$

where we have separated the field into intensity and phase according to  $A(t, z) = \sqrt{I(t, z)} \exp[-i\varphi(t, z)]$  and assumed the signal is stationary to set  $t = 0$ . In order to calculate the average in (8), we make use of the joint probability functions for chaotic and Gaussian-amplitude fluctuating light [11], given by (9), shown at the bottom of the page, for chaotic fluctuating light and

$$p_g(I(\tau), I(0)) = \frac{\exp[-(I(\tau) + I(0) - 2r\sqrt{I(\tau)I(0)})/2\bar{I}(1 - r^2)]}{8\pi\bar{I}\sqrt{I(\tau)I(0)}(1 - r^2)} \quad (10)$$

for Gaussian-amplitude fluctuating light. Here we have dropped the  $z$  dependence of the intensity and phase for simplicity (that is,  $I(\tau) = I(\tau, 0)$ ),  $r$  is given by  $r = e^{-\gamma\tau/2}$ , and  $\bar{I}$  is the mean intensity. The input optical spectra have a Lorentzian lineshape with a linewidth (full width at half maximum, FWHM) given by  $\gamma$ . The average in (8) is calculated by numerically integrating the joint probability distribution functions for chaotic and Gaussian-amplitude fluctuating light [(9) and (10)] multiplied by the terms bracketed in (8). The optical spectrum is then calculated by fast Fourier transformation of the correlation function.

Fig. 1 shows the output optical spectra from the SOA at several different values of  $\alpha$  for (a) chaotic (thermal) input light and (b) Gaussian-amplitude input light (with no phase fluctuation). The spectrum of the intensity-smoothed output broadens considerably as  $\alpha$  increases beyond a value of 1.0, less than the typical value for many SOA's ( $\alpha = 4-6$ ). Also, the Gaussian-amplitude input light results in a much broader output optical spectrum than does chaotic light of the same input linewidth. The output spectra are slightly broadened for  $\alpha = 0$  in both cases of input light. This is consistent with the results presented in [3] for chaotic fluctuating input light smoothed by a dye amplifier. Fig. 2 shows the FWHM linewidths ( $\Delta\omega$ ) of the spectra presented in Fig. 1 scaled by the input linewidth ( $\gamma$ ) versus  $\alpha$  and illustrates the broadening of output spectra with increasing amplitude-to-phase coupling.

The spectral broadening and differences between the two cases of input light can easily be understood by considering that when an intensity fluctuation is smoothed a corresponding fluctuation occurs in the carrier density. Since a change in the carrier density causes a change in the phase proportional to  $\alpha$ , when intensity noise is smoothed phase noise is generated by an amount that increases with  $\alpha$ . The differences in the

$$p_{ch}(I(\tau), \varphi(\tau), I(0), \varphi(0)) = \frac{\exp[-\{I(\tau) + I(0) - 2r\sqrt{I(\tau)I(0)} \cos(\varphi(\tau) - \varphi(0))\}/\bar{I}(1 - r^2)]}{4\pi^2\bar{I}^2(1 - r^2)} \quad (9)$$

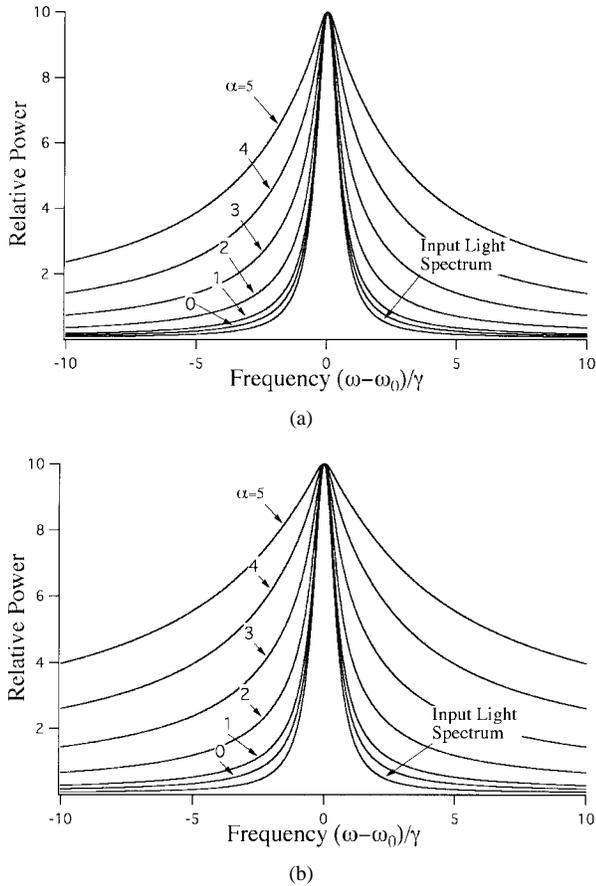


Fig. 1. Calculated input optical spectra and SOA output spectra for (a) chaotic fluctuating input and (b) Gaussian-amplitude input light with  $\alpha$  ranging from 0 to 5. The optical spectra are normalized to have the same peak power. The frequency is scaled to the linewidth (FWHM) of the input spectrum ( $\gamma$ ).

output spectra for the two input light models can be understood by considering that both probability distributions model light with the same input optical spectrum. The optical spectrum is the magnitude squared of the Fourier decomposition of the electric field and thus contains contributions from both the intensity and phase fluctuations. Since Gaussian-amplitude fluctuating light has a purely fluctuating intensity and constant phase, the output spectrum for Gaussian-amplitude input light will broaden more than chaotic light input, which has both fluctuating phase and intensity. This difference can be seen in the joint probability distributions shown in (9) and (10). If one sets the phases to zero in (9), it is clear that the joint probability distribution for the Gaussian-amplitude input light [(10)] has a broader distribution than that of the chaotic input light [(9)]. Thus, the Gaussian-amplitude input light has stronger intensity fluctuations than chaotic input light with the same optical spectrum and will experience more spectral broadening as it passes through the smoothing amplifier.

A way to view the increase in phase noise in the output of the SOA is to measure the fringe visibility  $V(\alpha)$  in the interference between the input light and the intensity-smoothed output light in an interferometer balanced so that both beams have equal average intensity [3]. The fringe visibility is a measure of the loss of relative coherence between the output

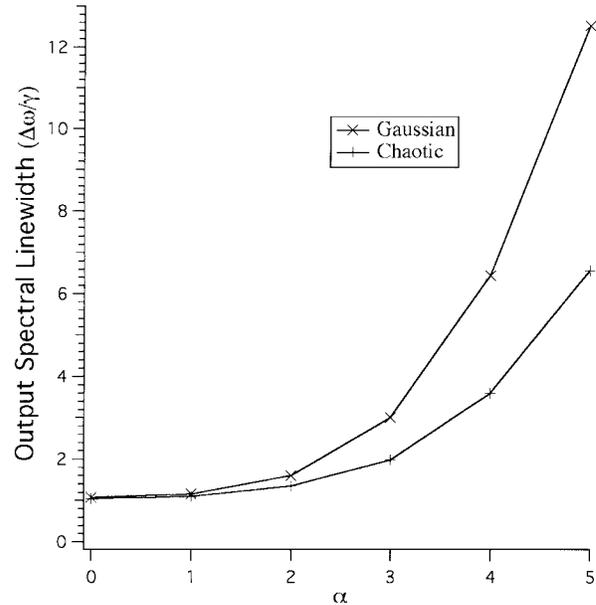


Fig. 2. Spectral linewidth (FWHM) of the calculated output of the SOA ( $\Delta\omega$ ) with chaotic fluctuating (+) and Gaussian amplitude fluctuating ( $\times$ ) input light as a function of  $\alpha$ . The linewidth is scaled by the linewidth of input light ( $\gamma$ ) and the lines connecting the data points are included as a guide to the eye.

signal and the input signal and is defined as

$$V(\alpha) = \frac{|A(t,0)A^*(t,L)|}{[|A(t,0)|^2|A(t,L)|^2]^{1/2}}. \quad (11)$$

We calculated the fringe visibility by substituting (7) into (11) and using (9) and (10) to calculate the average for the case of  $\tau = 0$  and  $\beta L \gg 1$ , resulting in

$$V(\alpha) = \sqrt{\frac{\alpha}{\sinh(\pi\alpha/2)}} \quad (12)$$

for Gaussian-amplitude fluctuating light and

$$V(\alpha) = \sqrt{\frac{\pi(1+\alpha^2)}{4\cosh(\pi\alpha/2)}} \quad (13)$$

for chaotic (thermal) fluctuating light. Fig. 3 shows the fringe visibilities as a function of  $\alpha$  in the cases of (a) chaotic and (b) Gaussian-amplitude input light as calculated using (12) and (13). As expected from the optical spectra shown in Fig. 1, the fringe visibility decreases with increasing alpha, and the fringe visibility for Gaussian-amplitude input light is lower than that for chaotic input light. In both input-light cases, the fringe visibility is less than 0.1 at a value of alpha that is typical for many SOA's ( $\alpha = 4$  to 6). This behavior is in contrast to the high fringe visibility measured in [3] for a dye amplifier, for which  $\alpha \cong 0$ .

In conclusion, we have shown that intensity smoothing of noisy optical signals with a semiconductor optical amplifier carries a penalty of spectral broadening (increased phase noise) that increases with increasing alpha parameter. This is shown in the broadening of the calculated output optical spectra and decreasing fringe visibility with increasing alpha. This penalty also increases with increasing strength of the

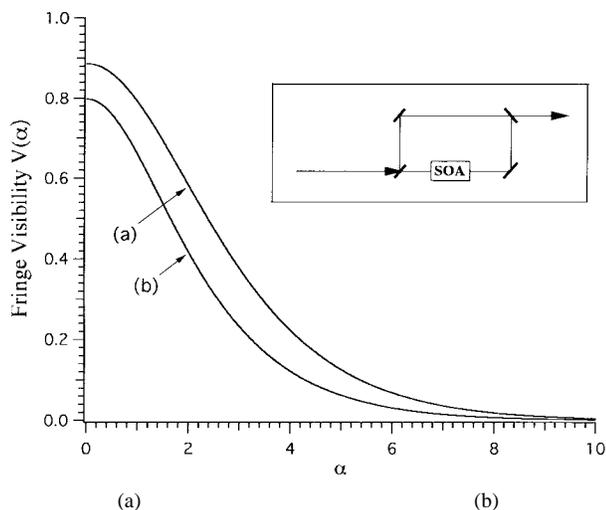


Fig. 3. Calculated fringe visibility of the interference between the amplifier input and output light for (a) chaotic fluctuating and (b) Gaussian-amplitude fluctuating input light as a function of  $\alpha$ . The inset shows the setup assumed in the calculation.

intensity fluctuations that are smoothed, as is illustrated by the higher spectral broadening and lower fringe visibility in the case of Gaussian-amplitude input light when compared to chaotic input light. This linewidth broadening may limit the performance of smoothing amplifiers in linewidth-sensitive systems.

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