Ultrafast photon-number correlations from dual-pulse, phase-averaged homodyne detection

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We propose and demonstrate a method for determining the two-time photon-number correlations of an optical field on ultrafast time scales. The method, which uses dual-pulse, phase-averaged, balanced-homodyne detection, is sensitive at the single-photon level and can have a quantum efficiency approaching 100%. Using this method we have determined the two-time, photon-number correlations on subpicosecond time scales of emission from a semiconductor optical amplifier. [S1050-2947(97)51003-1]

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Optical-field correlations contain information about the quantum properties of light. The two-time, photon-number correlation function $\langle :\hat{n}(t)\hat{n}(t+\tau):\rangle$ (with normal operator ordering) is an important example whose normalized form, referred to as second-order coherence, or $g^{(2)}(t,t+\tau)$, has been measured for both classical (e.g., photon-bunched [1]) and nonclassical (e.g., photon-antibunched [2]) states of light. Standard methods for determining two-time correlation statistics involve jointly counting the photons that arrive in two time windows at $t\pm\delta t/2$ and $t+\tau\pm\delta t/2$. These methods are presently limited by the time resolution and quantum efficiency (QE) of available detectors. Photon counting using photomultipliers or avalanche photodiodes (APDs) can achieve a time resolution and sampling window limited to approximately 10 ps, while photon-counting streak cameras have a demonstrated sampling time of approximately 20 ps [3]. Also, photocathode emission-type detectors such as photomultipliers and streak cameras can detect single photons (with 10–20 % QE) but cannot distinguish between n and $n+1$ photons for n greater than around 10 [3]. APDs operating in Geiger mode can detect single photons (with QE around 80%) but are saturated by a single photon and therefore cannot distinguish between $n=1$ and any higher number [4].

Nonlinear-optical mixing and an integrating, slow detector can be used to measure an intensity autocorrelation of the form $\int\langle :\hat{n}(t)\hat{n}(t+\tau):\rangle dt$ [5]. This gives a measurement on ultrafast time scales of intensity correlations, but only in the difference variable τ. This is appropriate for stationary sources but fails to capture the complete two-time correlation statistics of time-varying fields. Also, nonlinear techniques typically have low quantum efficiency, which degrades the information available in some cases.

Recently a method has been developed for determining the photon-number statistics within a *single* sampling window using pulsed, phase-averaged, balanced-homodyne detection (BHD) [6,7]. This technique is a derivative of optical homodyne tomography [8], a method by which phase-sensitive BHD is used to reconstruct the full quantum state of an optical field in a single space-time mode. These methods have a sampling time limited only by the duration of a reference, local oscillator (LO) pulse, and allow photodiodes with QE near 100% [9] to be used.

In this Rapid Communication we present a technique to determine the *two-time*, photon-number correlations from *dual-pulse*, phase-averaged BHD. The technique is related to recently proposed methods for determining the full quantum state of a two-mode optical field using BHD with a LO comprised of two, nonoverlapping pulses [10–12]. Opatrny, Welsch, and Vogel [12] have recently shown that certain two-time, photon-number correlations can be obtained from dual-pulse BHD, without first performing a full state reconstruction, by a method that requires the relative phase between the two LO pulses to be varied in a controlled manner. In contrast, the technique presented and demonstrated here uses only LO pulses with random phases and thus minimizes the amount of data required to obtain the photon-number correlations. To demonstrate our method we have performed experiments using thermal-like light from a semiconductor optical amplifier and LO pulses that yield a 150-fs sampling window.

A balanced-homodyne detector optically interferes the signal field with a strong LO on a 50-50 beam splitter whose outputs are detected using high-QE photodiodes. The interference amplifies a single-photon signal to a level much greater than the equivalent electronic noise of the photodiode [8]. The detector photocurrents are time-integrated and subtracted to give a measurement of the field quadrature amplitude, $q_{1\theta}$ [8,13]. The operator for the measured quadrature is $\hat{q}_{1\theta}=(\hat{a}_1e^{-i\theta}+\hat{a}_1^\dagger e^{i\theta})/2^{1/2}$, where θ is the phase of the LO and $\hat{a}_1$ is the annihilation operator for the signal that is in the same spatial-temporal (nonmonochromatic) mode as the LO:

$$\hat{a}_1\propto\int_0^T dt\ f_{\mathrm{LO}}^*(t-t_1)\hat{E}_S^{(+)}(t). \tag{1}$$

Here $f_{\mathrm{LO}}(t)$ is the normalized temporal-mode function for the LO pulse, which is centered at t_1. The detection integration time T is assumed to be long compared to the pulse durations. We have defined $\hat{E}_S^{(+)}(t)$ as the part of the (positive-frequency) signal-field operator that is in the spatial mode defined by that of the LO. If we assume that the LO pulse is much shorter than the inverse optical bandwidth of the signal, then Eq. (1) can be approximated as $\hat{a}_1\simeq K\hat{E}_S^{(+)}(t_1)$, where $K\propto\int_0^T dt\ f_{\mathrm{LO}}^*(t-t_1)$. For an LO with a known temporal mode (with $K\neq 0$) the complex constant K can be determined. Under these conditions measuring $q_{1\theta}$ provides a sampling of the signal field in a short time window.

The probability density $P(q_1,\theta)$ for the field quadrature amplitude $q_{1\theta}$ is built up by making repeated measurements while controlling or monitoring the LO phase θ. By using a LO whose phase is uniformly randomized over the range $[0,2\pi]$ the phase-averaged quadrature distribution is built up, $\overline{P}(q_1)=(1/2\pi)\int_0^{2\pi}d\theta\, P(q_1,\theta)$. Although all phase information is lost, the photon-number distribution can be determined by averaging certain sampling functions over this single distribution [6].

To extend this method to the two-time photon-number statistics requires that the signal field be sampled jointly in two separate time windows. If it were possible to separate temporally the field on ultrafast scales, we could send the two sections of the field to two separate BHDs. Using independent ultrashort LOs for each BHD we could then simultaneously sample the field in each of the two sections to measure the joint quadrature distribution $P(q_1,\theta;q_2,\beta)$. Here q_1 and q_2 are values of $\hat{q}_{1\theta}$ and $\hat{q}_{2\beta}$—the quadrature operators associated with the signal-field temporal modes that are selected by the first and second LO pulses having phases θ and β, respectively. Alternatively, if phase-random LOs are used, the phase-averaged distribution

$$\overline{P}(q_1,q_2)=1/(2\pi)^2\int_0^{2\pi}\int_0^{2\pi}P(q_1,\theta;q_2\beta)d\theta\, d\beta$$

is measured. From this the joint photon-number distribution can be obtained [10]. If instead only certain photon-number correlations are desired, it can be shown that the factorial moments of the joint photon-number distribution can be determined by

$$\langle\hat{n}_1^{(j)}\hat{n}_2^{(k)}\rangle=\langle\hat{a}_1^{\dagger j}\hat{a}_1^{j}\hat{a}_2^{\dagger k}\hat{a}_2^{k}\rangle=\int\int dq_1dq_2\overline{P}(q_1,q_2)\times\left[2^{j+k}\binom{2j}{j}\binom{2k}{k}\right]^{-1}H_{2j}(q_1)H_{2k}(q_2), \quad (2)$$

where $\hat{a}_2\approx K\hat{E}_S^{(+)}(t_2)$, $H_{2j}(q_1)$ are Hermite polynomials, and $\langle\ \rangle$ represents a quantum expectation value. This result is a generalization of the single-mode case treated by Richter [14].

Because it is not possible in practice to separate the signal field into two sections on ultrashort time scales it is not possible to measure directly either of the joint distributions, $P(q_1,\theta;q_2,\beta)$ or $\overline{P}(q_1,q_2)$. This can be circumvented by using a LO that is in a variable superposition of the two modes of interest [10–12]. Consider the case in which the two modes have the same spatial structure but with temporal modes chosen as two, localized pulses with shape $f(t)$, which are separated by a delay (t_2-t_1) that is greater than their durations, i.e.,

$$f_{\rm LO}(t)=e^{i\theta}\cos(\alpha)f(t-t_1)+e^{i\beta}\sin(\alpha)f(t-t_2). \quad (3)$$

Here α is an adjustable parameter setting the relative amplitude between the two LO pulses, while θ and β are their phases. Using this dual-pulse LO in homodyne detection with an integration time much greater than t_2-t_1 gives a measurement of a quadrature variable $\hat{Q}_{\alpha\theta\beta}$ that is a linear combination of the quadrature operators for the individual space-time modes; i.e., $\hat{Q}_{\alpha\theta\beta}=\hat{q}_{1\theta}\cos\alpha+\hat{q}_{2\beta}\sin\alpha$. By measuring the quadrature distribution $P(Q;\theta,\beta,\alpha)$ of the $\hat{Q}_{\alpha\theta\beta}$ variable for a sufficient set of discrete values of θ, β, and α, one can obtain the joint density matrix in the Fock basis [10]. In the case where phase-random LOs are used, the phase-averaged distribution

$$\overline{P}(Q,\alpha)=(1/4\pi^2)\int_0^{2\pi}\int_0^{2\pi}d\theta\, d\beta\, P(Q,\theta,\beta,\alpha)$$

is measured, from which the joint photon-number distribution for the two modes can be obtained.

Now we show that it is possible to obtain the factorial moments $\langle\hat{n}_1^{(j)}\hat{n}_2^{(k)}\rangle$ directly from the phase-averaged distribution, $\overline{P}(Q,\alpha)$, without a full reconstruction of the photon-number distribution. We make the ansatz that the factorial moments can be expressed as some linear combination of even moments of $\overline{P}(Q,\alpha)$, i.e.,

$$\langle\hat{n}_1^{(j)}\hat{n}_2^{(k)}\rangle=\sum_{l=0}^{l_{\rm max}}\sum_{i=0}^{l}C_{l,i}\int dQ\, Q^{2l}\overline{P}(Q,\alpha_{i,l}), \quad (4)$$

with a set of discrete values $\alpha_{i,l}=i\pi/2l$ (with $\alpha_{i,0}\equiv0$ when $l=0$), and $C_{l,i}$ represents a matrix of coefficients that will be determined. By rewriting $\overline{P}(Q,\alpha)$ as a projection integral [10]

$$\overline{P}(Q,\alpha)=\int\int dq_1dq_2\overline{P}(q_1,q_2)\,\delta(Q-q_1\cos\alpha-q_2\sin\alpha), \quad (5)$$

Eq. (4) becomes

$$\langle\hat{n}_1^{(j)}\hat{n}_2^{(k)}\rangle=\int\int dq_1dq_2\overline{P}(q_1,q_2)\times\sum_{l=0}^{l_{\rm max}}\sum_{i=0}^{l}C_{l,i}\sum_{s=0}^{l}\binom{2l}{2s}(\cos\alpha_{i,l})^{2l-2s}\times(\sin\alpha_{i,l})^{2s}q_1^{2l-2s}q_2^{2s}, \quad (6)$$

where we have used the generalized binomial expansion, keeping only the even powers.

The desired coefficients $C_{l,i}$ can be obtained by equating the summation inside the integral in Eq. (6) to the scaled product of Hermite polynomials inside the integral in Eq. (2). Note that because the highest-order term in $H_{2m}(x)$ is x^{2m}, we can limit the summation over l in Eq. (6) to $l_{\rm max}=j+k$. We can then determine the matrix of coefficients $C_{l,i}$ by equating equal powers of $q_1^{2\mu}q_2^{2\nu}$ for all $\mu\leqslant j$ and $\nu\leqslant k$. To simplify this we solve for the vector C_L (with coefficients $C_{L,i}$), associated with the particular value of $l=L$, for each $L\leqslant j+k$ separately. Only terms with $\mu+\nu=L$ are then involved, so we equate equal powers of $q_1^{2L-2\nu}q_2^{2\nu}$ on the right-hand side of Eqs. (2) and (6) for all $\nu\leqslant L$; this leaves the following requirement on the C_L:

$$\sum_{i=0}^{L}(2L)!(\cos\alpha_{i,L})^{2L-2\nu}(\sin\alpha_{i,L})^{2\nu}C_{L,i}=D_\nu^{(Ljk)}, \quad (7)$$

where $D_\nu^{(Ljk)}=0$ unless $L-j\leqslant\nu\leqslant k$, for which

$$D_\nu^{(Ljk)}=\frac{2^{2L-(j+k)}(j!)^2(k!)^2}{(j+\nu-L)!(k-\nu)!}(-1)^{j+k-L}. \tag{8}$$

Equation (7) represents a set of $L+1$ linearly independent equations (one for each $0\leqslant\nu\leqslant L$) that can be written in matrix form and solved by standard Gauss-Jordon reduction. Doing this for all $L\leqslant j+k$ we can determine the matrix of coefficients $C_{l,i}$ required in order for Eq. (4) (with $l_{\max}=j+k$) to be valid. The required number of LO amplitude combinations is determined *a priori* by the theory and scales with the order of the desired correlation function.

If the LO pulses are short, and $t_1\neq t_2$, we can write the measured correlations as

$$\begin{aligned}\langle\hat{n}_1^{(j)}\hat{n}_2^{(k)}\rangle &\simeq|K|^{2(j+k)}\\ &\times\langle[\hat{E}_S^{(-)}(t_1)]^j[\hat{E}_S^{(+)}(t_1)]^j[\hat{E}_S^{(-)}(t_2)]^k[\hat{E}_S^{(+)}(t_2)]^k\rangle,\end{aligned} \tag{9}$$

where t_1 and t_2 are the times at which the two LO pulses sample the signal field, respectively. This generalizes the single-mode case, which allows one to measure the factorial moment $\langle\hat{n}_1^{(j)}\rangle\approx|K|^{2j}\langle[\hat{E}_S^{(-)}(t_1)]^j[\hat{E}_S^{(+)}(t_1)]^j\rangle$.

As an important example consider $j=k=1$ for which the solution to Eq. (7) for $L=0,1,2$ leads to the result

$$\begin{aligned}\langle:\hat{n}(t_1)\hat{n}(t_2):\rangle=&\tfrac{2}{3}\langle Q^4\rangle_Q|_{\pi/4}-\tfrac{1}{6}\langle Q^4\rangle_Q|_0-\tfrac{1}{6}\langle Q^4\rangle_Q|_{\pi/2}\\ &-\tfrac{1}{2}\langle Q^2\rangle_Q|_0-\tfrac{1}{2}\langle Q^2\rangle_Q|_{\pi/2}+\tfrac{1}{4},\end{aligned} \tag{10}$$

where $\hat{n}_i=\hat{a}_i^\dagger\hat{a}_i=\hat{n}(t_i)$ is the operator associated with the number of photons that would be counted in an effective time window that is centered at t_i and whose width is proportional to $|K|^2$. In Eq. (10), $\langle\ \rangle_Q|_\alpha$ represents a Q average over the phase-averaged distribution $P(Q,\alpha)$, holding α fixed. To obtain Eq. (10) we used the fact that, because the two modes are required to be independent ($t_1\neq t_2$), their operators commute. In this example one needs to measure the quadrature distribution $P(Q,\alpha)$ for only three different values of α corresponding to the LO photons residing all in the first mode ($\alpha=0$), half in each of the two modes ($\alpha=\pi/4$), and all in the second mode ($\alpha=\pi/2$). If, in addition, $\langle\hat{n}(t_1)\rangle$ and $\langle\hat{n}(t_2)\rangle$ are determined from the single-pulse-LO technique [6], one can obtain the two-time second-order coherence defined (for a quasimonochromatic field) as [15]

$$g^{(2)}(t_1,t_2)=\frac{\langle:\hat{n}(t_1)\hat{n}(t_2):\rangle}{\langle\hat{n}(t_1)\rangle\langle\hat{n}(t_2)\rangle}. \tag{11}$$

This function is independent of QE.

We have demonstrated this method to determine the ultrafast two-time photon-number correlation statistics of a 4-ns optical pulse. The LO pulses are derived from a Ti:sapphire-based laser system that generates ultrashort, near transform-limited pulses (150 fs) at a wavelength of 830 nm and a repetition rate of 1 kHz. The experimental setup is shown in Fig. 1. Using a Mach-Zehnder interferometer the pulse is initially split in two and recombined to produce a dual-pulse LO. The pulse amplitudes are set equal (each pulse containing approximately 10^6 photons), using half-wave plates WP1 and WP2 in conjunction with the polarizing beam splitter PBS2. The two pulses are vertically and horizontally polarized, respectively, and are then combined with the signal pulse using PBS3. The action of WP3 and PBS3 allows us to vary the relative amplitude of each pulse in the LO according to $\cos\alpha$ and $\sin\alpha$, respectively. Two delay arms allow the LO pulses to be independently delayed relative to the signal pulse. The signal and LO fields are interfered, split, and balanced by WP4 and PBS4 and are detected by Si photodiodes (Hamamatsu S4280, 90% quantum efficiency, response time $\sim$1 ns). The current pulses from the two photodiodes are independently integrated, amplified, and sampled by the computer with two 16-bit analog-to-digital channels to yield two photoelectron numbers whose difference is scaled by the LO shot-noise level to give a field quadrature measurement [8].

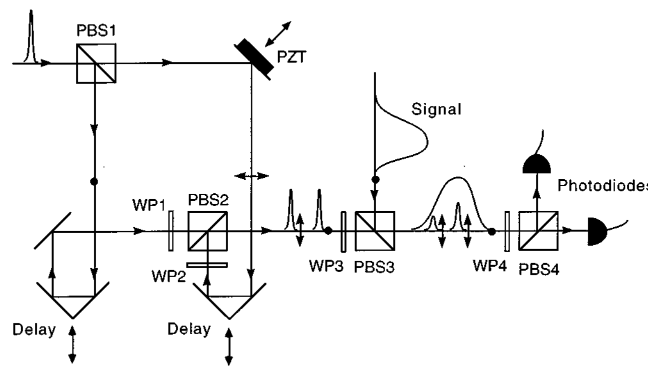

FIG. 1. Experimental setup. Beam polarization is indicated by an arrow (horizontal) or a filled circle (vertical).

Our signal is from a single-spatial-mode superluminescent diode (SLD) manufactured at the David Sarnoff Research Center [16]. The SLD is pumped by a voltage pulse that is triggered by a digital delay generator whose trigger is derived from the Ti:sapphire laser system. This trigger method allows us to synchronize the SLD optical pulse with the LO optical pulses with approximately 80-ps trigger jitter, which sets our overall time-resolution limit. The sampling time is 150 fs, set by the LO pulse. The output of the SLD is dominated by amplified spontaneous emission, which can be characterized as chaotic or thermal-like light [17]. The broadband emission at 830 nm is spectrally filtered with an interference filter to produce a 4-ns pulse having a 0.22-nm spectral width. The inverse bandwidth of the signal is approximately 3.4 ps, thereby validating the approximation leading to Eq. (9).

The signal and the LO pulses do not share a constant phase relationship because they come from separate sources. A piezoelectric translator in one arm of the interferometer is driven to randomize the relative phase ($\beta-\theta$) between the two LO pulses in order to validate the phase-random analysis scheme in Eq. (10). One LO delay arm was set so the center of the first LO pulse (t_1) occurred near the maximum of the signal pulse while the other LO delay arm was automated with a computer-controlled stepper motor allowing the relative delay between the two pulses ($\tau\equiv t_2-t_1$) to be stepped over a 20-ps range. For each value of τ we measured the quadrature distribution $P(Q,\alpha)$ for three different values of α: 0, $\pi/2$, and $\pi/4$. From these we computed $\langle:\hat{n}(t_1)\hat{n}(t_1+\tau):\rangle$. To normalize this correlation function as

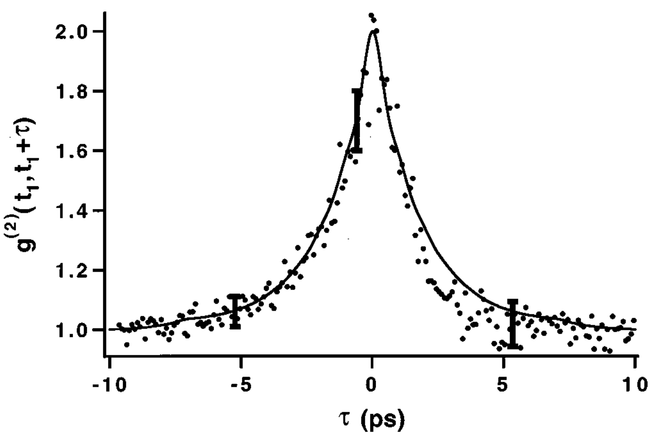

FIG. 2. The second-order coherence experimentally determined via BHD (dots) and from the measured optical spectrum (solid line). The value of t_1 is set to occur near the maximum of the signal pulse. Typical predicted statistical error bars are shown [18].

in Eq. (11), note that $\overline{P(Q,0)}$ and $\overline{P(Q,\pi/2)}$ are single-mode quadrature distributions. With the single-mode theory we thus determined $\langle \hat{n}(t_1)\rangle$ and $\langle \hat{n}(t_1+\tau)\rangle$, which have a value of approximately 8 photons that is constant over the 20-ps range.

The results for $g^{(2)}(t_1,t_1+\tau)$ are shown in Fig. 2. The second-order coherence decays from a value of 2 to a value of 1, which is expected for a thermal-like field. Because this method is limited to values of τ larger than a few LO pulse durations, we determined the value for zero delay, $g^{(2)}(t_1,t_1)$, by the single-LO pulse analysis [6]. We also measured $g^{(2)}(t_1,t_1+\tau)$ as a function of t_1 for a few values of τ. In each case its value remained constant as t_1 was varied over the 4-ns pulse, so we can say that the signal field is quasistationary over its duration. Thus the 80-ps trigger jitter between the signal pulse and the LO pulses, which sets the time-resolution limit by effectively averaging over time t_1, did not adversely affect the results shown in Fig. 2. The spread in the data is consistent with predicted statistical error bars [18].

To test the accuracy of the results obtained above we have measured the second-order coherence function by a second method that only applies to thermal light and that yields the time-integrated $g^{(2)}(\tau)=\int g^{(2)}(t_1,t_1+\tau)dt_1$. Because our signal field is quasistationary the time-integrated $g^{(2)}(\tau)$ will have the same τ dependence as $g^{(2)}(t_1,t_1+\tau)$. Using a scanning monochromator we measured the normalized optical spectrum $S(\nu)$ of the signal field. For thermal light its Fourier transform $\tilde{S}(\tau)$ can be used to calculate $g^{(2)}(\tau)=1+|\tilde{S}(\tau)|^2$ [15]. The result of this calculation is shown as the solid line in Fig. 2 and agrees quite well with the results of our more general technique, thereby verifying the thermal-like nature of our source.

We have proposed and demonstrated a technique by which two-time, photon-number correlations can be obtained on ultrafast time scales. The technique's high quantum efficiency and ultrashort sampling times offer promise for characterizing weak, ultrafast optical sources such as molecular and semiconductor systems.

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