Compound-cavity laser modes for arbitrary interface reflectivity

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Received April 19, 1993

Calculations of the longitudinal mode functions for the electric field of a compound-cavity laser show that the loss and gain coefficients are given by the Airy formula, as a function of the mode frequencies. Further, when the external cavity is longer than the gain-medium cavity, the frequency dependence of the gain is stronger than that of the loss, even when the intracavity surfaces are antireflection coated. These results provide a generalization of the oft used dielectric-bump model, in which the intracavity reflectivity has a lower bound fixed by the index mismatch at the interface between the gain medium and the external cavity.

The behavior of coupled-cavity lasers (CCL's) depends critically on the strength of coupling between the cavities.[1] An example of this dependence is in the statistics of the mode turn-on time following a sudden rise above threshold in a dye CCL.[2] Another example is in a semiconductor CCL in which coherence collapse, with its accompanying broadened spectrum, occurs when the external cavity length is large or the pump rate is high.[3] Finally, the mode-locking dynamics in a semiconductor CCL depends on whether the intracavity surface has high or low reflectivity.[4] The use of an antireflection coating on this surface is a common way to suppress self-pulsations (relaxation oscillations) and related instabilities.[5] The eigenfrequencies, spatial structures, mode competition, and gains and losses of the CCL modes all depend on the value of the intracavity surface reflectivity.[6] Despite the importance of modeling this class of laser, the modes of a CCL with antireflection-coated intracavity surface were not previously reported.

We present calculations of the longitudinal spatial mode functions for the electric field in a compound-cavity laser system in which the two outer mirrors are highly reflective. The effects of the functional form of the mode functions on the laser gain and loss are discussed. Our solution is a generalization of existing models[6,7] that allows one to treat an antireflection coating self-consistently at the surface coupling the cavities. We emphasize the case in which gain-medium length L_A and external-cavity length L_B are much longer than the wavelength of the laser emission. In this case we find that the mode normalization constants may be expressed by optical Airy formulas as functions of the mode frequencies and laser length. Thus the mode-gain line shape, which is proportional to the normalization constants, is also given by the Airy formula. This result alleviates the need for numerically solving a transcendental equation to determine the gain and loss coefficients, as is done in Ref. 6.

Figure 1 is a schematic of the compound, coupled-cavity system that we model. The longitudinal z dependence of the dielectric constant $\epsilon(z)$ is given by

$$\epsilon(z) = \begin{cases} n^2 & -L_A \le z \le 0 \\ {n_0}^2 & 0 \le z \le d \\ 1 & d \le z \le L_B + d \end{cases}, \tag{1}$$

where n and n_0 are the index of the refraction of the gain medium and of the dielectric layer with thickness d, respectively. When the output-coupler reflectivity $\mathfrak{R}$ is near unity the spatial mode functions for the compound cavity have the (one-dimensional) form

$$u_q(z) = \begin{cases} A_q \sin[k_q n(z + L_A)] & -L_A \le z \le 0 \\ C_q \exp(ik_q n_0 z) + \text{c.c.} & 0 \le z \le d \\ B_q \sin[k_q(z - L_B - d)] & d \le z \le L_B + d \end{cases}, \tag{2}$$

where the propagation constant k_q is related to the mode eigenfrequency ω_q and wavelength λ_q by $k_q = \omega_q/c = 2\pi/\lambda_q$. The quarter-wavelength-thick ($d = \lambda/4n_0$) dielectric layer of index n_0 allows the reflection to be different from the Fresnel reflection at the gain-medium–air interface. In the presence of the dielectric layer, the intensity reflectivity $\mathfrak{R}_i$ of this layer is related to n and n_0 by[8]

$$\mathfrak{R}_i = (J-1)^2/(J+1)^2, \tag{3}$$

where $J = {n_0}^2/n$. One can show this easily by removing the mirrors in Fig. 1 and considering a wave

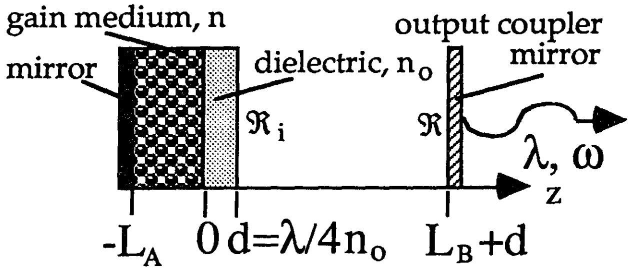

Fig. 1. CCL system.

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incident upon the gain-medium side, partially reflected from the dielectric with the remainder transmitted through the dielectric. When $n_0 = n(J = n)$, the usual Fresnel result is obtained. A perfect antireflection coating is modeled with $n_0 = \sqrt{n}(J = 1)$—in this case $\Re_i = 0$.

Expressions for the normalization constants A_q and B_q are found by applying the boundary conditions for the electric and magnetic fields given by classical electrodynamics and demanding orthonormality of the spatial mode functions $u_q(z)$. The boundary conditions are that the spatial mode functions $u_q(z)$ and their first derivatives $\mathrm{d}u_q(z)/\mathrm{d}z$ be continuous across the two dielectric interfaces shown in Fig. 1. After eliminating C_q, one obtains

$$A_q{}^2 = \frac{2}{nL_B}\left\{\frac{nL_A}{L_B}\left[1 - \frac{\sin(2k_q nL_A)}{2k_q nL_A}\right] + \frac{J\sin^2(k_q nL_A)}{\cos^2(k_q L_B)}\left[1 - \frac{\sin(2k_q L_B)}{2k_q L_B}\right] + \frac{d}{nL_B}\sin^2(k_q nL_A) \times \left[1 + \frac{nJ\sin^2(k_q nL_A)}{\cos^2(k_q L_B)}\right] + \frac{2}{nJk_q L_B}\cos(k_q nL_A)\sin(k_q nL_A)\right\}^{-1}, \tag{4}$$

$$B_q = -\sqrt{nJ}\,\frac{\sin(k_q nL_A)}{\cos(k_q L_B)}\,A_q = -\sqrt{\frac{J}{n}}\,\frac{\cos(k_q nL_A)}{\sin(k_q L_B)}\,A_q, \tag{5}$$

with the constraint that the k_q satisfy the eigenvalue equation

$$J\tan(k_q L_B) = \cot(k_q nL_A). \tag{6}$$

Figure 2 shows the functions $A_q{}^2$ and $B_q{}^2$ of Eqs. (4) and (5) plotted versus mode frequency for various values of $\Re_i$ for $n = 1.39$, $L_A = 0.01$ cm, and $L_B = 2.5$ cm. The functions are evaluated only at frequencies that satisfy Eq. (6), obtained numerically. Note that $A_q{}^2$ is modulated much more strongly than is $B_q{}^2$ in this case, where $nL_A \ll L_B$.

One can simplify the expressions for A_q and B_q dramatically when nL_A and L_B are much greater than λ_q by dropping terms proportional to d/L_B and $1/k_q L_{A,B}$. Then, to good approximation, Eqs. (4) and (5), when evaluated at the discrete frequencies given by Eq. (6), may be written as Airy formulas:

$$A_q{}^2 = \frac{2}{L_B}\left[\frac{\alpha_E J/n}{1 + \alpha_E(J^2 - 1)\sin^2(k_q nL_A)}\right], \tag{7}$$

$$B_q{}^2 = \frac{2}{L_B}\left[1 - \frac{\alpha_E JnL_A/L_B}{1 + \alpha_E(J^2 - 1)\sin^2(k_q nL_A)}\right], \tag{8}$$

where $\alpha_E = (1 + JnL_A/L_B)^{-1}$. Equations (7) and (8) are found by combining Eq. (6) and the identity $\cos^2(y) = [1 + \tan^2(y)]^{-1}$ with Eqs. (4) and (5).

For a long external cavity ($JnL_A \ll L_B$) we have $\alpha_E \approx 1$, so that

$$A_q{}^2 \approx \frac{2}{L_B}\left[\frac{\sqrt{F + 1}/n}{1 + F\sin^2(k_q nL_A)}\right], \tag{9}$$

$$B_q{}^2 \approx \frac{2}{L_B}\left\{1 - \frac{nL_A}{L_B}\left[\frac{\sqrt{F + 1}}{1 + F\sin^2(k_q nL_A)}\right]\right\}. \tag{10}$$

The coefficient of finesse F is given by $(J^2 - 1)$, which evaluates to $4\sqrt{\Re_i}/(1 - \sqrt{\Re_i})^2$ as expected from Fig. 1 (with the output coupler removed). If $L_B \to \infty$, then it is useful to renormalize $A_q{}^2$ by replacing the normalization factor $2/L_B$ by $2/L_A$ in relation (9), resulting in an expression derived by Kastler[9] for the on-axis emission spectrum of a Fabry–Perot étalon filled with an internal light source (except for the +1 in the numerator radical, which corresponds to the "background radiation" that Kastler neglected).

The orthonormality condition $n^2L_A A_q{}^2/2 + L_B B_q{}^2/2 = 1$ applies to $A_q{}^2$ and $B_q{}^2$ given at each level of approximation above.

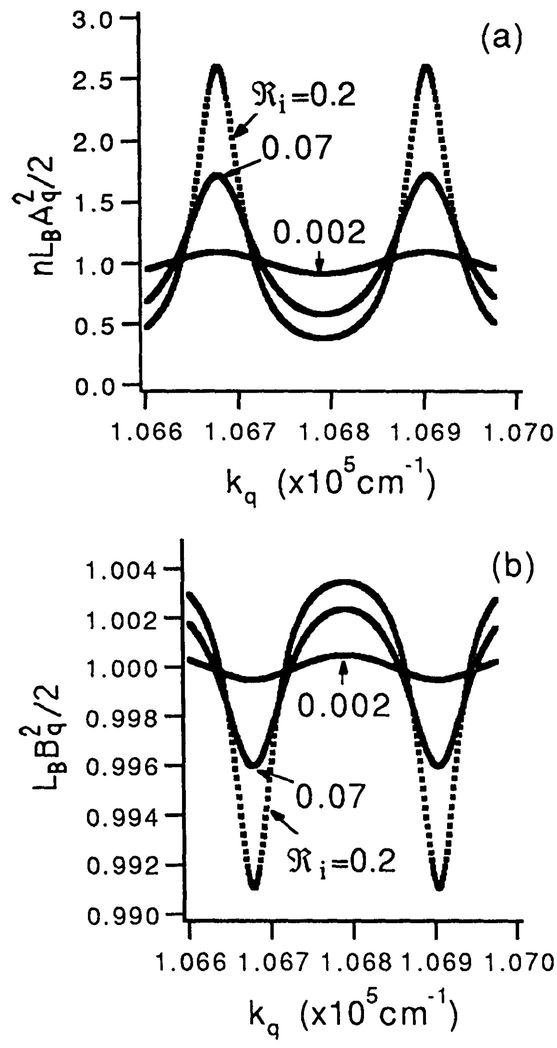

Fig. 2. Plot of the squared mode amplitude in the two parts of the cavity: (a) the thin gain medium, from Eq. (4), and (b) the long external cavity, from Eq. (5). Both are evaluated at the frequencies given by Eq. (6) for $n = 1.39$, $L_A = 0.01$ cm, and $L_B = 2.5$ cm.

In contrast to the present approach, the dielectric-bump model for compound-cavity laser systems (see Fig. 1) proposed by Spencer and Lamb,[10] and developed by Fader[11] and Chow,[12] models reflectivities $\mathcal{R}_i$ by replacing the dielectric layer (the region in Fig. 1 with an index of refraction n_0) with a delta-function layer (i.e., $d \rightarrow 0$). The dielectric constant $\epsilon(z)$ is then given by[6]

$$\epsilon(z) = n^2\theta(-z) + \theta(z) + (\eta/k)\delta(z), \qquad (11)$$

where η represents the strength of the intracavity reflectivity $\mathcal{R}_i$ and k is the average propagation constant for the electric field. If $\eta = 0$, then $\mathcal{R}_i$ is given by the gain-medium–air interface reflectivity:

$$\mathcal{R}_i = (n-1)^2/(n+1)^2 \qquad (\eta = 0). \qquad (12)$$

If $n = 1$, which is the case in Refs. 10–12, Fader showed that the layer reflectivity is given by

$$\mathcal{R}_i = \eta^2/(4+\eta^2) \qquad (n = 1). \qquad (13)$$

For $n \geq 1$, using the boundary conditions implied by Maxwell's equations, we find the general formula for the reflectivity in the dielectric-bump model to be

$$\mathcal{R}_i = [(n-1)^2 + \eta^2]/[(n+1)^2 + \eta^2]. \qquad (14)$$

Note that Eq. (14) reduces to the Fresnel formula corresponding to the gain-medium–air interface given by Eq. (12) when $\eta = 0$ and to Eq. (13) when $n = 1$. Thus $\mathcal{R}_i$ less than that given by Eq. (12) cannot be obtained from the dielectric bump model.

Rose *et al.*[6] imply that the dielectric-bump model can model antireflection coatings where $\mathcal{R}_i$ is less than that given by Eq. (12). Since they use the formula, Eq. (13), in the calculations of the longitudinal mode amplitudes, A_q^2 and B_q^2, their results cannot actually be applied to an antireflection-coated interface.

The laser gain for the $u_q(z)$ mode is proportional to A_q^2 by means of stimulated emission, and the loss, with the output coupler on the empty-cavity side, is proportional to $(1 - \mathcal{R})B_q^2$.[6,13–15] The result is that the dominant effect of coupling a short laser to a much longer empty cavity is through a frequency-dependent gain and not through a frequency-dependent loss, as is often assumed.[15] That this is so was also shown by Rose *et al.*[6] for intracavity reflectivities larger than that given by the Fresnel formula for the gain-medium–air interface. We find that this result is also valid in the important case that the intracavity surface is antireflection coated. When the output coupling is only through the external cavity, the loss coefficients are only weakly modulated in frequency. When the output coupling is through both ends of the laser, the modulation of gain with frequency is always greater than the modulation of the loss coefficients (as long as the external cavity length is longer than that of the gain medium). This contrasts with the method of modeling the laser dynamics by replacing the output coupler reflectivity with the product of that reflectivity and an Airy formula to represent the frequency-dependent loss.[15] In that method the gain is independent of mode frequency. The results presented here and in Ref. 6 suggest that the opposite is true: the dominant effect of the coupled-cavity geometry is a frequency modulation of the gain.

Valuable discussions with M. Rose and L. Hollberg are gratefully acknowledged. This research was supported in part by the National Science Foundation. M. Munroe was supported by a U.S. Department of Education Fellowship and S. Hodges was supported by a graduate fellowship from Santa Barbara Research Center.

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References

1. R. W. Tkach and A. R. Chaplyry, J. Lightwave Technol. **4,** 1655 (1986).
2. S. E. Hodges, M. Munroe, D. Adkison, W. Gadomski, and M. G. Raymer, Opt. Lett. **17,** 931 (1992).
3. C. W. Henry, in *Coherence, Amplification and Quantum Effects in Semiconductor Lasers,* Y. Yamamoto, ed. (Wiley, New York, 1991), pp. 55–57.
4. J. Palaski and K. Y. Lau, Appl. Phys. Lett. **59,** 7 (1991).
5. C. E. Wieman and L. Hollberg, Rev. Sci. Instrum. **62,** 1 (1991).
6. M. Rose, M. Lindberg, W. Chow, S. Koch, and M. Sargent III, Phys. Rev. A **46,** 603 (1992).
7. D. Marcuse and T. Lee, IEEE J. Quantum Electron. **QE-20,** 166 (1984).
8. M. Born and E. Wolf, *Principles of Optics* (Pergamon, Oxford, 1980), pp. 61–63.
9. A. Kastler, Appl. Opt. **1,** 17 (1962).
10. M. B. Spencer and W. E. Lamb, Phys. Rev. A **5,** 884, 893 (1972).
11. W. J. Fader, IEEE J. Quantum Electron. **QE-21,** 1838 (1985).
12. W. W. Chow, IEEE J. Quantum Electron. **QE-22,** 1174 (1986).
13. K. Shimoda, *Introduction to Laser Physics* (Springer-Verlag, Berlin, 1986), pp. 178–185.
14. M. Sargent III, M. O. Scully, and W. E. Lamb, *Laser Physics* (Addison-Wesley, Reading, Mass., 1977), pp. 96–110.
15. G. P. Agrawal and N. K. Dutta, *Long-Wavelength Semiconductor Lasers* (Van Nostrand Reinhold, New York, 1986), pp. 338–346.