Delay-time statistics of cooperative emission in the presence of homogeneous line broadening

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We have developed a theoretical model that allows us to study the delay-time statistics of superfluorescent pulses for the case of collisional broadening with a Lorentzian line shape. Our model is based upon a more robust definition of delay time, based upon an energy rather than an intensity reference. We calculate the probability distribution of delay times in the initiation regime of superfluorescence, where the inversion is essentially constant. We find that, as the collisional dephasing rate is increased, the mean delay time increases and the distribution broadens. We compare our results with those for the case of inhomogeneous broadening with a Lorentzian line shape and find the unexpected result that the statistical behavior is identical.

I. INTRODUCTION

This paper addresses the quantum-statistical properties of cooperative light emission from a collection of inverted atoms in the presence of various line-broadening mechanisms. It is known that both homogeneous and inhomogeneous broadening can inhibit the cooperative growth of a macroscopic dipole moment. However, it has not previously been established how the statistical properties of the emitted light depend on the mechanism responsible for the broadening. For example, it is not obvious if the statistical properties depend on whether the dephasing is due to collisional or to Doppler broadening.

Even in the absence of dephasing processes, there are large fluctuations in the shapes of the pulses emitted to the superfluorescence (SF) process.[1] The fluctuations are a macroscopic manifestation of quantum noise, which is responsible for initiating the superfluorescence.[2] In the case of pure superfluorescence (SF), it has been observed by Gibbs *et al.*[1] that the delay time, or buildup time with respect to the time of inverting the medium, fluctuates from shot to shot. This behavior was explained by Haake *et al.*[3] using a linearized, quantum-mechanical theory which includes propagation effects. They calculated the probability distribution of delay times, $P_I(\tau)$, defined such that $P_I(\tau)d\tau$ is the probability that the pulse intensity J first reaches a chosen reference intensity I in the time interval τ to $\tau+d\tau$. They found that the delay-time distribution broadens and shifts toward larger τ with increasing linewidth for the case of inhomogeneous line broadening with a frequency distribution that decreases faster than Lorentzian in the wings. They also obtained the somewhat puzzling result that, in the case of a Lorentzian-shaped inhomogeneous frequency distribution, $P_I(\tau)$ as defined above does not exist mathematically. It was not clear from their treatment whether this difficulty would also occur for the more physically realistic case of a homogeneously broadened Lorentzian line.

The purpose of the present paper is to extend the theory of SF delay-time statistics to this very case. This case is of particular interest because the influence of homogeneous dephasing on the SF process was studied in a recent experiment.[4] As in the case of inhomogeneous broadening, we find that the probability distribution $P_I(\tau)$ is ill-defined for a homogeneously broadened Lorentzian line. We therefore introduce an improved definition of delay time, based on integrated energy rather than on intensity. It is found that in the linear-gain regime, where the atoms remain essentially inverted, homogeneous and inhomogeneous broadenings lead to identical SF statistics if they both have Lorentzian line shapes.

In the linear-gain regime, the equations of motion for SF are identical in form to those governing stimulated Raman scattering.[5–7] Therefore, the present results can also be used directly to understand the buildup statistics of transient stimulated Raman scattering (SRS) in the presence of line broadening.

II. EQUATIONS OF MOTION

Let us consider a collection of $N=\rho AL$ inverted two-level atoms, contained in a cylindrical region of length L and cross-sectional area A, where ρ is the number density of excited atoms. We assume that the Fresnel number $A/\lambda L$, where λ is the transition wavelength, is close to unity. In such a case, it is a good approximation to ignore transverse effects and describe the propagation problem in terms of one-dimensional equations. For simplicity, we measure the longitudinal dimension in units of the length L of the interaction region so that $z=0$ and $z=1$ correspond to the two ends of the region. Furthermore, we measure time in terms of dimensionless time t, which is scaled by the standard superfluorescent decay time[8]

$$\tau_R=\frac{8\pi}{3}\frac{A}{\lambda^2}\frac{\tau_0}{N}\ , \tag{1}$$

where τ_0 denotes the natural lifetime of an isolated atom. For convenience we also introduce the local dimensionless time for each atom, which in terms of our scaled coordinates is given by $\tau=t-z/v$, where $v=c\tau_R/L$ is

the velocity of light in our dimensionless units. We assume that the sample is excited by "swept excitation" and is totally inverted at local time $\tau=0$. We calculate the time evolution of the initially inverted system using a fully quantum-mechanical approach. We describe the optical field in terms of an electric field operator whose slowly varying amplitude corresponding to the creation of photons is denoted by $\hat{E}^-(z,\tau)$, and describe the atomic polarization in terms of a collective field operator whose part corresponding to the lowering operator is represented by $\hat{R}^-(z,\tau)=[\hat{R}^+(z,\tau)^\dagger]$. We choose these field operators to be dimensionless. In particular, the physical electric field is measured in units of $\pi\rho L d\omega/c$ so that the number of emitted photons is given by $(N/4)\int E^2 dt$. Similarly, $\langle \hat{R}^- \rangle$ is taken to be the mean dipole moment per unit volume scaled by $id\rho/2$ where d is the atomic dipole transition moment.

Since we are interested primarily in the initiation of superfluorescence, we assume that the atomic inversion remains essentially constant (and equal to $+1$) for all times under consideration. The dynamical evolution of the system is then described by the linearized Maxwell-Bloch equations, including collisional damping:[7,8]

$$\frac{\partial}{\partial z}\hat{E}^-(z,\tau)=\hat{R}^+(z,\tau)\ , \tag{2a}$$

$$\frac{\partial}{\partial \tau}\hat{R}^+(z,\tau)=-\Gamma\hat{R}^+(z,\tau)+\hat{E}^-(z,\tau)+\hat{F}^+(z,\tau)\ , \tag{2b}$$

where Γ is equal to the damping rate multiplied by τ_R. Within this linearization approximation, the polarization field operators obey commutation relations appropriate to a Bose field, which imply that the second-order correlation function is given by[7]

$$\langle \hat{R}^+(z,0)\hat{R}^-(z',0)\rangle=\frac{4}{N}\delta(z-z')\ . \tag{3}$$

The mean value $\langle \hat{R}^+(z,0)\rangle$ is equal to zero. The Langevin noise operators $\hat{F}^\pm(z,\tau)$ are required by the fluctuation-dissipation theorem, and obey the relations[5]

$$\langle \hat{F}^-(z,\tau)\rangle=\langle \hat{F}^+(z,\tau)\rangle=0\ , \tag{4a}$$

$$\langle \hat{F}^+(z,\tau)\hat{F}^-(z',\tau')\rangle=\frac{4}{N}\delta(z-z')2\Gamma\delta(\tau-\tau')\ . \tag{4b}$$

We assume that $\hat{R}^\pm$ and $\hat{F}^\pm$ both obey the Gaussian decorrelation relation for higher-order correlation functions. The solution to the Maxwell-Bloch equations (2) is well known and is given by[6,9]

$$\hat{E}^-(z,\tau)=\int_0^z dz' K_1(z-z',\tau)\hat{R}^+(z',0)+\int_0^z dz'\int_0^\tau d\tau' K_2(z-z',\tau-\tau')\hat{F}^+(z',\tau')\ , \tag{5a}$$

where

$$K_1(z-z',\tau)=e^{-\Gamma\tau}I_0([4(z-z')\tau]^{1/2})\ , \tag{5b}$$

$$K_2(z-z',\tau-\tau')=e^{-\Gamma(\tau-\tau')}I_0([4(z-z')(\tau-\tau')]^{1/2})\ , \tag{5c}$$

where I_j denotes the jth-order modified Bessel function. In writing the solution in the form shown, we have omitted a term proportional to the initial value of the electric field operator $\hat{E}^-(0,\tau)$ since this term does not contribute to normally ordered correlation functions in the case in which the emission process is spontaneously initiated. Note that this solution predicts large amplification of the emitted field with increasing time τ. Consequently, at the output face of the interaction region ($z=1$), the emitted radiation will consist of a large number of photons and thus we hereafter describe the statistical properties of the generated radiation approximately by treating the field as a classical stochastic process, $\hat{E}^-(z,\tau)\rightarrow E^-(z,\tau)$ having Gaussian statistics.[3,7]

III. DELAY-TIME STATISTICS

One of the more dramatic signatures of SF is the presence of a long time delay between the excitation and the peak of the intensity of the emitted light. The delay time is known to display large statistical fluctuations even among systems that are prepared identically. In order to predict the statistical properties of the delay time, Haake *et al.*[3] have defined the delay time T as the interval between the time of excitation and the time $t(I)$ when the emitted intensity J first reaches some reference intensity I. In accordance with this definition, Haake *et al.* then estimate the probability density of the distribution of delay times using the formula

$$P_I(T)=\left\langle \delta(T-t(I))\Theta\left[\frac{dJ}{dt}\right]\right\rangle\ , \tag{6}$$

where Θ is the unit step function, and the brackets now designate a classical ensemble average. Note that if the intensity were a monotonically increasing function of time, there would be no need to include the unit step function in the definition (6) and in this case $P_I(T)$ would give exactly (rather than estimate) the probability density of first-passage times. Haake *et al.* have shown that whenever the underlying statistical process is Gaussian, Eq. (6) can be expressed explicitly in the form

$$P_I(T)=\frac{\langle \dot{J}(T)\rangle I}{2\langle J(T)\rangle^2}e^{-I/\langle J(T)\rangle}\times\left[1+\mathrm{erf}\left[\frac{\langle \dot{J}(T)\rangle^2 I}{4\langle J(T)\rangle\beta^2(T)}\right]^{1/2}\right]+\frac{I^{1/2}\beta(T)}{\pi\langle J(T)\rangle^3}\times\exp\left[-\frac{I}{\langle J(T)\rangle}-\frac{\langle \dot{J}(T)\rangle^2 I}{4\langle J(T)\rangle\beta^2(T)}\right]\ , \tag{7}$$

where

$$\langle J(T)\rangle=\langle E^-(T)E^+(T)\rangle\ ,$$

$$\beta^2(T)=\langle E^-(T)E^+(T)\rangle\langle \dot{E}^-(T)\dot{E}^+(T)\rangle-|\langle E^-(T)\dot{E}^+(T)\rangle|^2$$

and where the overdot indicates a derivative with respect to the argument.

Haake *et al.* have applied Eq. (7) to the calculation of delay-time statistics for the case of inhomogeneous line

broadening and have found that the formula leads to reasonable predictions for any assumed line shape except that of a Lorentzian (or one that falls off in the wings even more slowly), in which case the quantity $\langle \dot{E}^- \dot{E}^+ \rangle$ is divergent. When we try to evaluate Eq. (7) for the problem treated in this paper, namely that of homogeneous broadening in the impact approximation [i.e., Eqs. (2)] we find that we cannot obtain reasonable predictions either, because the quantity $\langle \dot{E}^- \dot{E}^+ \rangle$ diverges. One can show from Eq. (5) that the expression for $\langle \dot{E}^- \dot{E}^+ \rangle$ diverges because it involves a delta function evaluated at zero time argument. We believe that the physical origin of this divergence is the fact that $E^-(t)$ fluctuates rapidly and hence can cross the reference level $E_R = I^{1/2}$ infinitely many times, leading to a divergence in Eq. (6).

Although the emitted field shows rapid fluctuations, leading to the divergence problems just mentioned, the total energy emitted up to time t, defined as

$$w(t) = \frac{N}{4} \int_0^t dt' E^-(1,t') E^+(1,t') , \tag{8}$$

is a monotonically increasing function of time. In our units, $w(t)$ is simply the number of photons emitted up to time t. We therefore propose as a more robust estimate of the delay time T the time required for the emitted energy to reach some threshold energy level W. The probability density of delay times in accordance with this definition is hence given by

$$P_W(T) = \langle \delta(T - t(W)) \rangle . \tag{9}$$

Since each trajectory crosses the reference energy W exactly once, the function $P_W(T)$ constitutes a properly normalized probability density and T is a proper first-passage time. Using well-known properties of the delta function, this expression can be expressed as

$$P_W(T) = \left\langle \delta(W - w(T)) \frac{dw(T)}{dT} \right\rangle = -\frac{d}{dT} \int_{-\infty}^{\infty} \frac{d\xi}{2\pi i} \frac{1}{\xi} e^{i\xi W} \langle e^{-i\xi w(T)} \rangle , \tag{10}$$

which can be evaluated to give[10]

$$P_W(T) = \frac{d}{dT} \int_{-\infty}^{\infty} \frac{d\xi}{2\pi i} \frac{1}{\xi} \frac{\exp(i\xi W)}{\prod_j (1 + i\xi\lambda_j)} , \tag{11}$$

where the λ_j are the time-dependent eigenvalues of the electric field autocorrelation function

$$A(t_1,t_2) = \frac{N}{4} \langle E^-(t_1) E^+(t_2) \rangle \tag{12}$$

defined on the interval $0 \le t_1, t_2 \le T$. In the present context, eigenvalues λ_j and eigenfunctions $\phi_j(t)$ are defined to be solutions of the integral equation

$$\int_0^T dt_2 \, A(t_1,t_2) \phi_j(t_2) = \lambda_j \phi_j(t_1) .$$

Note that the eigenvalues and eigenfunctions depend on T, but we have suppressed this dependence in our notation. It is easy to show that

$$\mathrm{Tr}(A) = \sum_k \lambda_k = \frac{N}{4} \int_0^T \langle E^-(t) E^+(t) \rangle dt = \langle w(T) \rangle . \tag{13}$$

The Fourier integral in Eq. (11) can be evaluated easily to give the result

$$P_W(T) = \frac{d}{dT} \sum_k \frac{e^{-W/\lambda_k}}{\prod_{j \neq k} (1 - \lambda_j/\lambda_k)} \tag{14}$$

for the probability density function for the delay times. It is straightforward to verify that $\int_0^\infty P_W(T) dT = 1$.

An important special case of Eq. (14) occurs when only one of the eigenvalues is nonzero. As a consequence of Eq. (13), the single eigenvalue is then just equal to $\langle w(T) \rangle$. Equation (14) is straightforwardly evaluated for this case to give the simple result

$$P_W(T) = \frac{\langle J(T) \rangle W}{\langle w(T) \rangle^2} e^{-W/\langle w(T) \rangle} , \tag{15}$$

where $\langle J(T) \rangle = \langle \dot{w}(T) \rangle$ has been used. Physically, the case of a single dominant eigenvalue occurs when the collisional decay rate Γ of Eq. (2) vanishes.[10] In this case, one finds from Eqs. (5) that the mean emitted intensity is given by

$$\langle J(T) \rangle = \{ [I_0(2t^{1/2})]^2 - [I_1(2t^{1/2})]^2 \} . \tag{16}$$

In the general case where collisional dephasing effects are present, the correlation function $A(t_1,t_2)$ has more than one nonzero eigenvalue and to find these eigenvalues we diagonalize $A(t_1,t_2)$ numerically. From the solutions (5) it follows that the diagonal elements of A are given by

$$\begin{aligned} A(t,t) &= \langle J(t) \rangle \\ &= \{ e^{-2\Gamma t} [I_0^2(2t^{1/2}) - I_1^2(2t^{1/2})] \} \\ &\quad + 2\Gamma \int_0^t dx \, e^{-2\Gamma x} [I_0^2(2x^{1/2}) - I_1^2(2x^{1/2})] \end{aligned} \tag{17a}$$

and that the off-diagonal elements are given by

$$\begin{aligned} A(t_1,t_2) &= \frac{e^{-\Gamma(t_1+t_2)}}{t_2 - t_1} \\ &\quad \times \left[f(t_1,t_2) + 2\Gamma \int_0^{t_m} f(t_1 - x, t_2 - x) e^{2\Gamma x} dx \right] , \end{aligned} \tag{17b}$$

where t_m is the lesser of t_1 and t_2, and where

$$\begin{aligned} f(t_1,t_2) &= t_2^{1/2} I_0(2t_1^{1/2}) I_1(2t_2^{1/2}) \\ &\quad - t_1^{1/2} I_0(2t_2^{1/2}) I_1(2t_1^{1/2}) . \end{aligned} \tag{17c}$$

The eigenvalues thereby obtained are used in conjunction with Eq. (14) to obtain the probability density function $P_W(T)$.

In Fig. 1 the probability density function $P_W(T)$ is plotted as a function of the dimensionless delay time T for several values of the dimensionless dephasing rate Γ. For this graph, the reference energy W was taken to be 10^5 in our dimensionless units of emitted photons. Note that as the damping rate increases the probability distribution shifts and broadens dramatically to longer delay. The increase in the delay time is consistent with the pre-

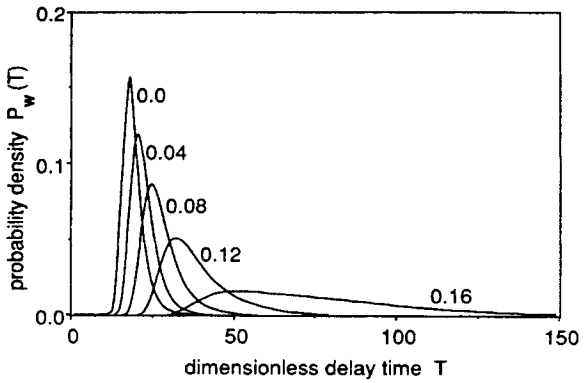

FIG. 1. Probability distribution of delay times based on a reference energy W, plotted for several different values of the dimensionless dephasing rate Γ.

diction of Schuurmans and Polder,[11] and with the experimental results of Okada *et al.*[12] and Malcuit *et al.*[4] We have found that for any value of the reference energy in the range 10^4-10^8 the distributions remain qualitatively similar to those shown in the figure.

In Fig. 2, we plot the mean delay time

$$\langle T\rangle=\int_0^\infty TP_W(T)dT$$

as a function of the dimensionless dephasing rate. Note that the mean delay time is a monotonically increasing function of Γ. Also plotted in the figure is the delay time of the mean energy, that is, the time at which the mean energy $\langle w(T)\rangle$ reaches the reference level $W=10^5$. The similarity of the two curves is a strong indication that our definition of delay time based on an energy reference provides a consistent basis for the statistical analysis of superfluorescent pulses.

IV. COMPARISON OF HOMOGENEOUS AND INHOMOGENEOUS BROADENING

We now have a robust definition of delay time which allows the analysis of delay-time statistics in the case of a homogeneously broadened, Lorentzian-shaped atomic line. It is thus of interest to question whether this definition also allows the analysis of the inhomogeneously broadened, Lorentzian-shaped atomic line, which, as mentioned above, could not be treated using the

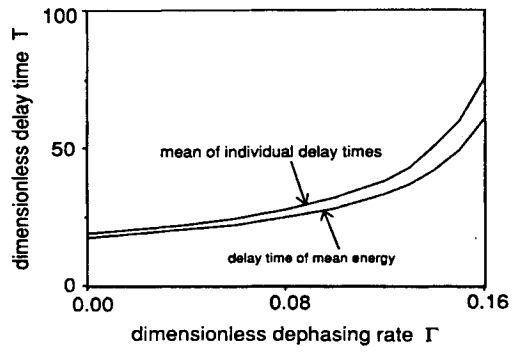

FIG. 2. Illustration of the increase of delay time with increasing dephasing rate.

definition of Haake *et al.* A related question is whether the resulting statistical properties depend only on the atomic line shape, or also on which type of broadening (homogeneous or inhomogeneous) is present.

In the case of inhomogeneous broadening, the equations of motion for cooperative emission in scaled variables are[3]

$$\frac{\partial}{\partial z}\hat{E}^-(z,\tau)=\int d\Delta\, p(\Delta)\hat{R}^+(z,\tau,\Delta)\ , \tag{18a}$$

$$\frac{\partial}{\partial \tau}\hat{R}^+(z,\tau,\Delta)=i\Delta\hat{R}^+(z,\tau,\Delta)+\hat{E}^-(z,\tau)\ , \tag{18b}$$

where $p(\Delta)$ is the normalized line-shape function and Δ is the frequency detuning of a group of atoms from the center of the distribution. The polarization operator of this group of atoms is $\hat{R}^+(z,\tau,\Delta)$, and in the swept-excitation case has the initial correlation properties

$$\langle\hat{R}^+(z,0,\Delta)\rangle=0\ , \tag{19a}$$

$$\langle\hat{R}^+(z,0,\Delta)\hat{R}^-(z',0,\Delta')\rangle=\frac{4}{N}\delta(z-z')\frac{1}{p(\Delta)}\delta(\Delta-\Delta')\ . \tag{19b}$$

The solution of Eq. (18) with swept-excitation initial conditions can easily be obtained using the methods in Ref. 3, giving

$$\hat{E}^-(z,\tau)=\int_0^z dz' K_1(z-z',\tau)\hat{S}^+(z',0)+\int_0^z dz'\int_0^\tau d\tau' K_2(z-z',\tau-\tau')\hat{G}^+(z'\tau')\ , \tag{20a}$$

where $\hat{S}^+$ is the frequency-averaged initial polarization

$$\hat{S}^+(z',0)=\int d\Delta\, p(\Delta)\hat{R}^+(z',0,\Delta)\ , \tag{20b}$$

and we have also defined the operator

$$\hat{G}^+(z',\tau')=\int d\Delta\, p(\Delta)(\Gamma+i\Delta)\hat{R}^+(z,0,\Delta)e^{i\Delta\tau'}\ . \tag{20c}$$

The kernal functions K_1 and K_2 are the same as in the solution (5) for the homogeneous-broadening case.

The solution (20) of the inhomogeneous equations is formally identical to the solution (5) of the homogeneous equations. So if we can equate $\hat{S}^+(z',0)$ with $\hat{R}^+(z',0)$, and also $\hat{G}^+(z',\tau')$ with $\hat{F}^+(z',\tau')$, we would have a complete equivalence between the two cases. This equivalence can indeed be shown in the statistical sense. First, note that all of the mentioned operators obey the Gaussian decorrelation relation for higher-order correlation functions, since the newly introduced operators $\hat{S}^+$ and $\hat{G}^+$ are homogeneous linear functionals of the operators $\hat{R}^+$ and $\hat{F}^+$. Second, note that the two-point correlation functions of the new operators are given by

$$\langle\hat{S}^+(z,0)\hat{S}^-(z',0)\rangle=\int d\Delta\, p(\Delta)\int d\Delta' p(\Delta')\times\langle\hat{R}^+(z,0,\Delta)\hat{R}^-(z',0,\Delta')\rangle=\frac{4}{N}\delta(z-z') \tag{21a}$$

and

$$\langle \hat{G}^{+}(z,\tau)\hat{G}^{-}(z',\tau')\rangle = \int d\Delta\, p(\Delta)\int d\Delta' p(\Delta')(\Gamma^2+\Delta^2)\langle \hat{R}^{+}(z,0,\Delta)\hat{R}^{-}(z',0,\Delta')\rangle e^{i\Delta\tau}e^{-i\Delta'\tau'}$$
$$= \frac{4}{N}\delta(z-z')\int d\Delta\, p(\Delta)(\Gamma^2+\Delta^2)e^{i\Delta(\tau-\tau')}\ , \tag{21b}$$

where Eq. (19b) has been used. By comparing Eq. (21a) with (3) we see that $\hat{S}^{+}(z,0)$ and $\hat{R}^{+}(z,0)$ have the same correlation functions (to all orders), and therefore these can be thought of as corresponding identical random processes (in the classical language). In order to evaluate Eq. (21b) we must specify the form of the inhomogeneous line shape $p(\Delta)$. If we assume it to be a normalized Lorentzian,

$$p(\Delta)=\frac{\Gamma/\pi}{\Delta^2+\Gamma^2}\ , \tag{22}$$

we immediately obtain

$$\langle \hat{G}^{+}(z,\tau)\hat{G}^{-}(z',\tau')\rangle = \frac{4}{N}\delta(z-z')2\Gamma\delta(\tau-\tau')\ . \tag{23}$$

Comparing Eq. (23) to Eq. (4), we see that $\hat{G}^{+}(z,\tau)$ and $\hat{F}^{+}(z,\tau)$ correspond to identical random processes. Note that in the case that $p(\Delta)$ is not a Lorentzian, $\hat{G}^{+}(z,\tau)$ will not be delta correlated in time, and thus the equivalence to $\hat{F}^{+}(z,\tau)$ is lost.

We conclude that for a Lorentzian line shape the generated electric field $\hat{E}^{-}(z,\tau)$ has identical statistical properties for both homogeneous and inhomogeneous broadening. Therefore, the same analysis of delay-time statistics given in Sec. III can be applied to the case of inhomogeneous broadening with a Lorentzian line shape, and the results are identical (Figs. 1 and 2). In the case that $p(\Delta)$ is not Lorentzian, the same analysis can still be used, but now the field autocorrelation function $\langle \hat{E}^{-}(t_1)\hat{E}^{+}(t_2)\rangle$ in Eq. (12) must be evaluated using solution (20). We will not do that here, although it is straightforward to do so.

V. CONCLUSIONS

We have introduced a new definition of delay time based on an energy reference rather than an intensity reference. This new definition is generally applicable, since it allows us to treat even the Lorentzian line shape which the former definition could not treat.

On the basis of our new definition of delay time, we have calculated how the probability distribution of delay times depends on the homogeneous dephasing rate Γ. We find that the delay time is a monotonically increasing function of the dephasing rate. Note that our new definition of delay time is an energy delivery time. In the experimental paper of Malcuit *et al.*,[4] delay time was taken to be a "turn-on" or rise time of the emitted radiation. In accordance with this definition, they find that the delay time first increases and then decreases with increasing dephasing rate.

We find that our probability distribution of delay times broadens dramatically for dephasing rates $\Gamma \geq 0.15$. This broadened distribution is indicative of the enhanced fluctuations that are characteristic of the transition from pure SF to amplified spontaneous emission (ASE). This value $\Gamma=0.15$ is close to the critical dephasing rate introduced by Schuurmans and Polder[11] to characterize the transition from SF to ASE. These authors estimate that the transition from SF to ASE occurs for a dipole dephase time of

$$T_2=(\tau_r\tau_D)^{1/2}\ ,$$

where τ_D is the mean delay time in the absence of dephasing processes. In our dimensionless units, $\tau_r=1$ and $\tau_D=19$, implying that the transition occurs near $\Gamma=0.23$.

We have furthermore demonstrated for the case of a Lorentzian line shape that homogeneous and inhomogeneous broadening lead to identical statistical properties for the SF pulses in the linear regime. It is likely that this equivalence between homogeneous and inhomogeneous broadening does not hold in the nonlinear regime of SF (i.e., when the depletion of the population inversion becomes important) nor in the linear regime for other line shapes.

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[1]For a review of the properties of the superfluorescence process, see, for instance, M. H. V. Schuurmans, Q. H. V. Vrehen, D. Polder, and H. M. Gibbs, in *Advances in Atomic and Molecular Physics*, edited by D. Bates and B. Bederson (Academic, New York, 1981), Vol. 17, p. 168.

[2]H. M. Gibbs, Q. H. V. Vrehen, and H. M. J. Hikspoors, Phys. Rev. Lett. **39**, 547 (1977).

[3]F. Haake, H. King, G. Schröder, J. W. Haus, and R. Glauber, Phys. Rev. A **20**, 2047 (1979); F. Haake, J. W. Haus, H. King, G. Schröder, and R. Glauber, *ibid.* **23**, 1322 (1981).

[4]M. S. Malcuit, J. J. Maki, D. J. Simkin, and R. W. Boyd, Phys. Rev. Lett. **59**, 1189 (1987); a more complete analysis of the results of this experiment including numerical solutions of the quantum-statistical equations describing the SF and ASE processes in the nonlinear regime is planned to be given in J. J. Maki, M. S. Malcuit, M. G. Raymer, R. W. Boyd, and P. D. Drummond (unpublished).

[5]J. Mostowski and M. G. Raymer, Opt. Commun. **36**, 237 (1981).

[6]M. G. Raymer and J. Mostowski, Phys. Rev. A **24**, 1980 (1981).

[7]T. von Foerster and R. J. Glauber, Phys. Rev. A **3**, 1984 (1971).
[8]F. T. Arecchi and E. Courtens, Phys. Rev. A **2**, 1830 (1970).
[9]R. Glauber and F. Haake, Phys. Lett. **68A**, 29 (1978).
[10]M. G. Raymer, K. Rzążewski, and J. Mostowski, Opt. Lett. **7**, 71 (1982).
[11]M. F. H. Schuurmans and D. Polder, Phys. Lett. **72A**, 306 (1979).
[12]J. Okada, K. Ikeda, and M. Matsuoka, Opt. Commun. **26**, 189 (1978); **27**, 321 (1978).