Quantum-state tomography of two-mode light using generalized rotations in phase space

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We introduce a method for quantum-state tomography for a pair of optical modes, which we call generalized rotations in phase space. Rather than using a dual-mode local oscillator field for implementing balanced-homodyne detection, as in previous methods, a single-mode local oscillator field can be used in our method, greatly simplifying certain aspects of experiments. The signal field is put through a series of general transformations before entering the detection system where quantum statistics are measured.

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Quantum-state tomography (QST) is comprised of a broad set of experimental methods for determining the quantum state of an ensemble of similarly prepared physical systems from a set of statistical measurements [1]. It has emerged as a powerful and useful tool in atomic [2], molecular [3], and optical physics [4]. QST can provide a complete statistical characterization of a system (in the sense that no further information can be acquired according to quantum mechanics). Because of this it can be used to characterize quantities which do not correspond to quantum observables, such as the phase of an optical field [5]. QST may play an important role in the testing and calibration of quantum information devices—for example, a state teleporter [6] or a quantum logic gate [7].

Most work to date has concentrated on determining states for a single degree of freedom, for example a single mode of the optical field [5]. But because quantum mechanics says interesting things about correlations that may exist between two degrees of freedom (Bell's inequalities, for example [8]), it is interesting to develop QST for more than one degree of freedom. A step in this direction was taken with a proposal for measuring the joint state of a pair of optical modes using balanced homodyne detection (BHD) in which the reference laser [local oscillator (LO)] field is formed into a linear combination of the two signal modes [9,10]. By effecting a series of linear transformations on the state of the LO field and making BHD measurements on the signal field for each of these various LO states, the two-mode state of the signal field can be determined. This ''dual-mode-LO'' method was used experimentally to determine two-time photon-number correlations in the case that the two modes of interest are in the form of two short pulses (''wave-packet modes'') having the same center frequency and polarization, but variable time delays [11].

The dual-mode-LO method was subsequently specialized to provide a method for the full characterization of the quantum state of polarization of a light beam [12]. In recent experimental work we have found that practical difficulties, involving the polarization dependence of the behavior of optical elements (such as beam splitters), make the dual-mode-LO scheme difficult to implement in this case. For this reason we propose an alternate method for two-mode QST, which we refer to as quantum-state tomography using generalized rotations in phase space (GRIPS).

In any state-reconstruction method, a set of linear transformations must be applied in order to generate a tomographically complete set of observables (or a quorum), whose statistical properties are measured. (For review, see [13].) In the original dual-mode-LO method, the set of transformations is applied to the two-mode state of the LO field, while in the new GRIPS method it is applied to the two-mode signal field. The main advantage of the GRIPS method is that only a single-mode field need be measured for each of the linear transformations in the complete set (rather than a two-mode field as in the original method). This is far easier experimentally since then a standard single-mode BHD setup can be used, and problems associated with the polarization dependence of optical elements do not arise in a strong manner.

The overall scheme is illustrated schematically in Fig. 1. The two signal modes of interest (1,2) together enter a linear signal-transforming device, to be described below. The device has two control knobs whose settings determine which of many linear transformations is applied to the signal modes. The output of the device consists of two separated fields (3,4). The relation between input-mode annihilation operators a_1, a_2 and output annihilation operators a_3, a_4 is

$$a_3 = \cos(\gamma/2) a_1 + \sin(\gamma/2)\exp(i\Delta\phi) a_2, \tag{1}$$

$$a_4 = -\sin(\gamma/2) a_1 + \cos(\gamma/2)\exp(i\Delta\phi) a_2, \tag{2}$$

where the relative-phase-shift parameter $\Delta\phi$ is controlled by knob A and varies between 0 and 2π, and the relative-amplitude parameter γ is controlled by knob B and varies between 0 and π. The transformation ensures the commutation relation $[a_3, a_4^\dagger] = 0$, meaning the two output modes are separately measurable. For our purposes, only a_3 need be measured; this levels a_4 as a potentially usable field for further experimentation. Mode a_3 is defined such that it is in the same mode as the LO. Mode a_3 enters the dc balanced-homodyne detector, which is comprised of a 50/50 beam splitter which linearly combines (interferes) mode a_3 and the LO mode having phase ϕ. The interfered beams are incident on photodetectors, which integrate the photon numbers incident during a chosen time interval. After subtraction and scaling, the quantity that is measured on a single trial is the ''combined quadrature amplitude'' [9,10],

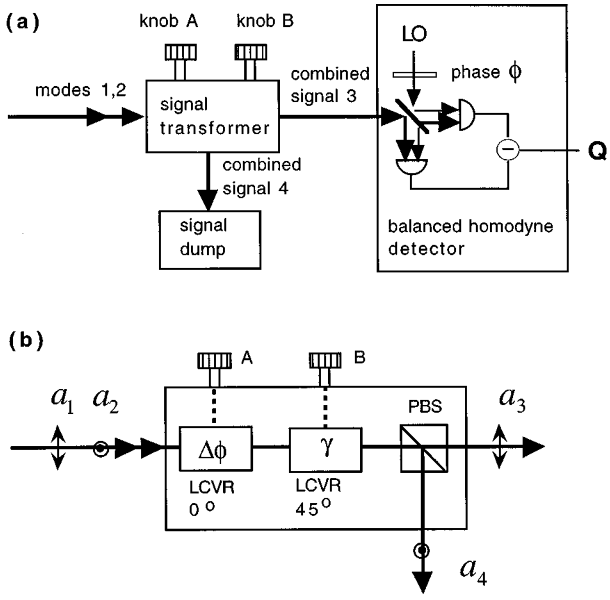

FIG. 1. (a) In the GRIPS method of two-mode state tomography, the input modes 1 and 2 are linearly combined with certain phases and amplitudes in a signal-transforming device, and the combined mode 3 is measured by a balanced homodyne detector with a single-mode local oscillator field. Knobs *A* and *B*, respectively, control the relative phase and amplitudes of the input-mode transformation. (b) The signal transformer for the case of measuring the quantum polarization state of an optical field. The two orthogonally polarized modes pass through a pair of voltage-controlled liquid-crystal variable phase retarders (LCVR) and then a polarizing beam splitter (PBS).

$$Q(\gamma/2,\phi,\Delta\phi)=[a_3\exp(-i\phi)+a_3^\dagger\exp(i\phi)]/2^{1/2}$$
$$=\cos(\gamma/2)q_1(\phi)+\sin(\gamma/2)q_2(\phi-\Delta\phi), \tag{3}$$

where the familiar single-mode quadrature operators are $q_i(\theta)=[a_1\exp(-i\theta)+a_1^\dagger\exp(i\theta)]/2^{1/2}$, for $i=1,2$.

The linear signal-transforming device, which implements the GRIPS, can be constructed in several alternate ways. In the case of polarization modes, perhaps the most convenient implementation is to use a pair of liquid-crystal voltage-controlled variable phase retarders, as shown in Fig. 1(b). The beam first passes through a retarder whose fast axis is parallel to the horizontal axis (defined, in this case, as the a_1 mode). This results in a controllable relative-phase shift $\Delta\phi$ between the vertically and horizontally polarized modes. Then the beam passes through the second retarder whose fast axis is set at 45° from the vertical direction and whose fast-slow phase shift is γ. Following this, the beam enters a polarizing beam splitter, which separates the beam into a horizontally polarized mode (a_3) and a vertically polarized mode (a_4). The horizontally polarized mode (a_3) is given by

$$a_3=\exp(i\gamma/2)\{a_1\cos(\gamma/2)$$
$$+a_2\sin(\gamma/2)\exp[i(\Delta\phi+\pi/2)]\}. \tag{4}$$

The overall phase-shift term $\exp(i\gamma/2)$ can be eliminated by adjusting the LO phase by $\gamma/2$. The shift in the relative phase term by $\pi/2$ can be corrected by redefining the zero relative-phase voltage for the liquid-crystal variable retarder. With these adjustments, Eq. (4) becomes identical to Eq. (1), and this completes the transformation for the polarization mode case. In the other case of modes comprised of two short pulses, the signal beam must be split into physically distinct beams and a relative time delay must be introduced between the two paths before recombining [14].

To characterize the quantum state of the field, it is sufficient to measure Q repeatedly and to thereby build up a histogram for fixed values of $\gamma,\Delta\phi,\phi$, which approximates the probability density $P'(Q;\gamma/2,\Delta\phi,\phi)$ to observe a value Q on a given trial. The measurements are repeated for different combinations of values of $\gamma,\Delta\phi,\phi$. The mathematical procedure for determining the quantum state from this set of measured histograms is the same as developed earlier for the dual-mode-LO detection scheme so the details will not be repeated here [15]. The result expresses the two-mode density matrix in the joint number basis as [9]

$$_1\langle n_1|_2\langle n_2|\rho|m_1\rangle_1|m_2\rangle_2$$
$$=\frac{1}{2}\int_{-\infty}^{\infty}dQ\int_0^{\pi}d\gamma\int_0^{\pi}d\Delta\phi\int_0^{2\pi}d\phi\; P'(Q;\gamma/2,\Delta\phi,\phi)$$
$$\times R_{n_1m_1}^{n_2m_2}(Q;\gamma/2,\Delta\phi,\phi), \tag{5}$$

where the ''sampling function'' is

$$R_{n_1m_1}^{n_2m_2}(Q;\gamma/2,\Delta\phi,\phi)=r_{n_1m_1}^{n_2m_2}(Q;\gamma/2)\exp[i(n_1+n_2$$
$$-m_1-m_2)\phi-i(n_2-m_2)\Delta\phi], \tag{6}$$

and $r_{n_1m_1}^{n_2m_2}(Q;\gamma/2)$ is a known function. (See [9] for graphed examples.)

From the measured density matrix, all statistical moments of any two-mode system operators can be deduced by straightforward calculation. In fact, if one prefers to obtain only certain field or number correlations (moments), it is not necessary to reconstruct first the full density matrix and then compute the moments. It is possible to obtain directly the desired moments by measuring quadrature data at only a small set of judiciously chosen values of $\gamma,\Delta\phi,\phi$ [16].

Finally we point out that the new GRIPS method may be applied to the characterization of the quantum polarization state of a light beam, which has been defined in terms of a certain subset (''polarization sector'') of the entire two-mode density matrix [12]. In that case the quadrature histograms are collected while randomizing the phase ϕ of the LO, since the overall phase is irrelevant to determining polarization properties. The relative-phase-shift parameter $\Delta\phi$ must be systematically stepped, and quadrature data collected for each setting, since $\Delta\phi$ corresponds to the phase between polarization components in the signal beam, which is important in determining polarization.

The method of the generalized rotations in phase space is currently being implemented in experiments in our laboratory to study the polarization properties of two-mode quadrature-squeezed light, as well as light emitted from optically pumped, vertical-cavity surface-emitting semiconductor lasers. It has come to our attention that a related method has recently been independently proposed by D'Ariano, Sacchi, and Kumar [17].

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